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HEATING AND WEAR OF AIRCRAFT BRAKES

WORK CARRIED OUT BY BRITISH MESSIER LTD
FOR MINISTRY OF SUPPLY UNDER CONTRACT
NO 6/STORES/16502/CB 20(A)

BY

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SUMMARY

The work covered by this memorandum was done by British Messier Ltd., for the Ministry of Supply, under Contract No. 6/Stores/16502/CB.20(a).

Temperatures at the surface of an aircraft brake drum or disc are determined for the case of a landing. An investigation is made into the effect of cooling, which is shown to be quite small in a representative case. Curves are given, by the aid of which peak temperatures may be rapidly determined.

Certain experimental results are quoted, which tend to show that friction lining wear is decreased by using brake discs of high conductivity material, and is increased by an increase in rubbing velocity, for a given energy input and torque. Various possible explanations of these phenomena are discussed, and the conjecture is put forward that an important factor is the temperature gradient near the surface of the friction material. It is shown that if this is the case, the favourable effect of high conductivity should be very much reduced if brakes are applied gradually, instead of suddenly, as is the practice during testing.

Suggestions are made as to experimental work, and a method is given by which the cooling coefficient can be measured.

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1. INTRODUCTION

1.1 Reasons for Enquiry

The object of the present enquiry was to obtain data required for the improvement of brake performance, particularly from the points of view of wear of friction material and weight and design of heat reservoir (disc or drum).

Experimental results obtained by British Messier Ltd. show that for a given energy input

(a) rubbing speed has an important effect on wear.

(b) wear of friction material can be reduced, sometimes substantially, by using brake discs made of high conductivity material instead of the usual steel.

(c) chrome plating the disc surfaces reduces the wear.

Calculations showed that both the above effects could take place while the peak surface temperatures remained substantially unaffected, and there was therefore no obvious way of accounting for them. It was felt that in order to shed light on the problem, it was necessary to make a comprehensive investigation of thermal phenomena in a brake, and to correlate this with findings on wear. This is what the present report endeavours to do.

1.2 General Assumption

In textbooks and papers on heat flow problems, the assumption is invariably made that the specific heat of metallic materials can be regarded as a constant. Fig.1. shows the actual values of the specific heat of steel and copper as a function of temperature, from which it appears that the above assumption may involve quite appreciable errors. Any other assumption, however, would lead to quite unmanageable complexities in the mathematical treatment, and hence in this report we shall follow the usual practice, and regard specific heat as a constant. In section 2.6 however, a method is given for obtaining a rough estimate of the error involved in this simplification, in certain cases.

Further and still less accurate assumptions are involved in the treatment of the cooling problem, and are discussed in detail in section 3.1.

For the above reasons, calculations based on the theory presented here should be regarded as strictly valid only for purposes of comparison. For numerical calculation of temperatures and thermal gradients, the results obtainable should be regarded as comparatively rough estimates only.

By "temperature" we shall mean the excess temperature (of the brake surface, or any other part) over that of the atmosphere.

1.3 Friction Phenomena and Heat Input

As is well known, the surfaces obtained in ordinary manufacturing practice show considerable departures from perfect flatness, or more generally from any nominal shape to which they may be machined, ground, etc. Due to this, contact between nominally "mating" surfaces of hard materials is limited to a number of small regions corresponding to high points on the surfaces; the actual area of contact is the sum of the areas at all such points.

The theory of contact and friction phenomena has been treated in considerable detail by Bowden and Tabor, (Ref.1.), who show that for metallic surfaces the actual contact area is given by P/f, where P is the normal load and f an effective crushing stress equal to about three times the yield stress of the softer material of the pair. Frictional heat is generated at the true contact regions only, and is therefore localised at a number of quite small areas. At some of these points of contact "junctions" are formed between the surfaces, possibly by a kind of welding process, and these junctions must be sheared for motion to proceed. At some of the junctions shearing takes place not at the junction itself but below it in one or the other surface, i.e. a small particle becomes detached from one surface, generally adhering to the other. This accounts for frictional wear.

Brake lining manufacturers have some reservations to make as to the validity of the above theory when applied to conditions obtaining in brakes. In a talk with the authors, Dr. Parker of Ferodo Ltd. expressed the view that in actual brakes intimate contact takes place over quite large areas and not over a large number of "points"; he did think however that in many cases contact was very far from being achieved over the whole "nominal" contact area, and that the actual contact area increased, (up to a point), with increasing contact pressure, there being an optimum pressure for which contact was practically uniform over the whole pad.

In any event it would appear to be common knowledge that below certain pressures at least contact is limited to a fraction of the pad area, and thus the rate of heat input may be far from uniform in a direction perpendicular to the sliding motion, i.e. radially on a drum, axially on a disc.

Such non-uniformity may be a major factor in brake wear phenomena in certain cases, and some of its possible effects will be discussed in section 5. To begin with however we shall consider the idealised case of a brake in which the rate of heat input is strictly constant over the whole surface.

2. SURFACE TEMPERATURE OF UNCOOLED BRAKES

2.1 Method of Calculation

A general method of calculating brake surface temperatures is given in Ref.2, under the assumption that all the heat generated flows into the brake drum of disc. Ref.2 also examines the effect of conduction of heat away from the drum into the wheel; this particular aspect of the problem will not be dealt with here, and is obviously irrelevant in the case of disc brakes.

The method of Ref.2 consists of determining the surface temperature for a constant rate of heat input, from which it is possible to deduce the temperature for any arbitrary heat input, given as a function of time. This method does not lend itself easily to being extended to deal with the case of a cooled brake.

An alternative method is given in Appendix II; this method has been worked out for the case of a brake subjected to a constant torque, and is not very easily extended to more complex conditions; it does however lend itself to extension to the case of a cooled brake.

Both methods involve certain mathematical approximations, and hence the results show certain differences, which are however insignificant for practical purposes.

A mathematically exact method of calculation for an infinitely thick drum or disc, again for the constant thoque case, is given in Ref.3.

It is shown in Ref.2 that surface temperature can be expressed as a function of the dimensionless time parameter

$$t = \frac{t_2}{t_1}$$

where t_1 = "relaxation time" = $\frac{cDh^2}{k}$

t_2 = time

c = specific heat		of material of
D = density		brake drum or
k = conductivity		disc

h = thickness of drum, or <u>half</u> thickness of disc.

Other significant parameters are

T = the duration of the stop

$a = \frac{t_1}{T} = \frac{cDh^2}{kT}$

H = the total heat input per face

Q = the instantaneous rate of heat input per face

A = the area of the drum or disc per face.

2.2 Surface Temperature for Gradual Application of Brake

In Ref.2 the brake surface temperature is worked out for an aircraft landing under the following conditions:

(i) Immediately after touch-down the aircraft is fully airborne.

(ii) Subsequently the lift varies as the square of the velocity.

(iii) The brake drag is proportional to the load on the wheels, i.e. to (weight minus lift), and therefore increases gradually from zero.

(iv) The ratio of aerodynamic drag to lift is equal to the ratio of brake drag to wheel load (coefficient of retardation), which is said to be a fair approximation to actual conditions.

It can then be shown that the rate of heat input into the brake is given by

$$Q = \frac{4H}{T}\left(\frac{2t_2}{T} - \frac{3t_2^2}{T^2} + \frac{t_2^3}{T^3}\right) \tag{1}$$

and that the surface temperature is given by:

(i) for $t < 1/3$

$$S = \frac{8hH}{TkA\sqrt{3}}\sqrt{t}\left(\frac{4at}{3} - \frac{8a^2t^2}{5} + \frac{16a^3t^3}{35}\right) \tag{2}$$

(ii) for $t > 1/3$

$$S = \frac{4hH}{TkA}\Bigg[- \left(\frac{a}{27} + \frac{a^2}{135} + \frac{a^3}{2268}\right) + t\left(\frac{2a}{3} + \frac{a^2}{9} + \frac{a^3}{135}\right)$$

$$+ t^2\left(a - a^2 - \frac{a^3}{18}\right) + t^3\left(-a^2 + \frac{a^3}{3}\right) + t^4\frac{a^3}{4}\Bigg] \qquad (3)$$

A temperature - time curve for a typical case (from Ref.2) is given in Fig.2.

2.3 Surface Temperature for Constant Torque Case

When the torque (or, more exactly, the retardation caused by the brake) is constant, the rate of heat input is given by

$$\dot{Q} = \frac{2H}{T}\left(1 - \frac{t_2}{T}\right) = \frac{2H}{T}(1 - at) \qquad (4)$$

In Appendix I, the resulting temperature is calculated by the method of Ref.2. The results are:

(i) for $t < 1/3$

$$S = \frac{4hH}{TkA\sqrt{3}}\sqrt{t}\left(1 - \frac{2at}{3}\right) \qquad (5)$$

(ii) for $t > 1/3$

$$S = \frac{2hH}{TkA}\left[t + \frac{2}{3} - a\left(\frac{t^2}{2} + \frac{5t}{9} - \frac{1}{54}\right)\right] \qquad (6)$$

An alternative treatment is given in Appendix II, giving slightly different results due to differences in certain mathematical approximations used in the calculations. It is shown in Appendix II that the surface temperature is given by

$$S = \frac{2hH}{TkA}\left[\phi_1(t) - a\,\phi_2(t)\right] \qquad (7)$$

where $$\phi_1(t) = (t + \tfrac{1}{2})\,\mathrm{erf}\sqrt{t} + \frac{1}{\sqrt{\pi}}\sqrt{t}\,e^{-t} \qquad (8)$$

$$\phi_2(t) = \tfrac{1}{2}\left[t(t+1) - \tfrac{1}{4}\right]\mathrm{erf}\sqrt{t}$$

$$+ \frac{1}{2\sqrt{\pi}}\sqrt{t}\cdot(t + \tfrac{1}{2})\,e^{-t} \qquad (9)$$

For $t < 1$, these expressions approximate to

$$\phi_1(t) = \frac{2}{\sqrt{\pi}}\sqrt{t}\left(1 + \frac{t}{3}\right) \qquad (10)$$

$$\phi_2 (t) = \frac{4}{3\sqrt{\pi}} t^{3/2} (1 + \frac{t}{5}) \qquad (11)$$

For t > 1, the expressions approximate to

$$\phi_1 (t) = t + \tfrac{1}{2} \qquad (12)$$

$$\phi_2 (t) = \tfrac{1}{2}\left[t (t + 1) - \tfrac{1}{4} \right] \qquad (13)$$

For large values of a, corresponding to small values of t and to a thick disc, substitution of (10) and (11) into (7) gives

$$S \approx \frac{4hH}{TkA\sqrt{\pi}} \sqrt{t} \left(1 - \frac{2at}{3}\right) = \frac{4H}{TA\sqrt{\pi cDK}} \sqrt{t_2} \left(1 - \frac{2t_2}{3T}\right) \qquad (14)$$

which agrees with the exact expression derived in Ref.3 for an infinitely thick drum or disc, and is also identical with (5) apart from the substitution of $\sqrt{\pi}$ for $\sqrt{3}$.

A typical disc brake, (made by British Messier Ltd.), consists of one disc of high conductivity material 0.9" thick and 11.1" x 6.5" diameter. The total energy input is 1.5×10^6 ft. lb. and the stopping time 15 seconds. For the brake disc material c = 0.10 cals/gram/degree C, k = 0.8 cals/cm./degree C/sec., D = 9.0 grams/cu.cm. The resulting temperature-time relationship is given by curve A, Fig.3.

2.4 Peak Temperature of Uncooled Brakes

As shown in Appendix VI, the peak surface temperature for gradual application of brakes under the conditions assumed in section 2.2 is given by

$$S_{max_1} = \frac{H}{AcDh} F_1 (a) \qquad (15)$$

and for constant torque conditions of section 2.3 by

$$S_{max_2} = \frac{H}{AcDh} F_2 (a) \qquad (16)$$

For small values of the parameter a $\left(= \frac{t_1}{T} = \frac{\text{relaxation time}}{\text{stopping time}} \right)$

$F_1 (a) \approx F_2 (a) \approx 1$, and the peak temperature is practically equal to that which would be obtained if the disc was infinitely thin, or its conductivity was infinite, or the run was infinitely long.

$F_1 (a)$ and $F_2 (a)$ are plotted against a in Fig.4.

It is also shown in Appendix VI that for large values of the parameter a, equations (15) and (16) can be replaced by

$$S_{max_1} = \frac{1.18H}{A\sqrt{T}} \times \frac{1}{\sqrt{cDk}} \qquad (17)$$

$$S_{max_2} = \frac{1.07H}{A\sqrt{T}} \times \frac{1}{\sqrt{cDk}} \tag{18}$$

In this case the disc or drum behaves as if it were infinitely thick.

For numerical calculations, it is useful to note that $\frac{H}{A}$ cals/sq. cm. $= \frac{E}{20}$, where E is the energy input in ft-lb. per square inch.

2.5 Critical Thicknesses

(i) From Fig.4 it is apparent that S_{max} is practically constant up to $a < 0.2$. Thus for $a < 0.2$

$$S_{max} \approx \frac{H}{AcDh} \tag{19}$$

and a disc or drum may be regarded as "thin".

Now $a = \frac{cDh^2}{kT}$, and therefore $h = \sqrt{\frac{kaT}{cD}}$

For steel, c = 0.117, k = 0.115, D = 7.8 in c.g.s. units.

For the high conductivity material (H.C.M.)

c = 0.1, k = 0.8, D = 9.0

Thus for a = 0.2, for steel h = 0.159$\sqrt{T}$ cm = 0.062 $\sqrt{T}$ inches and for the high conductivity material h = 0.408$\sqrt{T}$ cm = 0.16$\sqrt{T}$ inches. With T = 16 seconds, h = 0.248" for steel, 0.64" for H.C.M. (we recall that h is the total thickness for a drum, the <u>half</u> - thickness for a disc).

(ii) For the gradually applied load case considered in section 2.2, when a > 1 the maximum temperature is given to a sufficient degree of accuracy by equation (17), and the brake behaves as if it was infinitely thick. This corresponds to h = 0.14$\sqrt{T}$ for steel and h = 0.36$\sqrt{T}$ for high conductivity material (H.C.M.) or, for T = 16 seconds, h = 0.56" for steel and h = 1.44" for other material. In this case no appreciable decrease in peak surface temperature may be obtained by increasing the disc or drum thickness.

(iii) For the constant torque case, the peak temperature does not approach the "infinitely thick" values until "a" is quite large, but it may still be said that the figures just quoted are a limit beyond which further additions of metal are comparatively ineffective.

Quite generally, the factors $F_1(a)$ and $F_2(2)$ express the ratio of the actual peak surface temperature to the peak average temperature (i.e. ratio of total heat to thermal capacity), and are therefore inversely proportional to the efficiency of the heat reservoir, if the latter is defined as the ratio of the temperature which would be obtained if the heat was uniformly distributed throughout the metal to the peak surface temperature actually obtained.

If we take 16 seconds as a near approximation to the stopping times, we may say:

> **For steel brakes the metal is used most efficiently if the thickness does not exceed 0.25" for a drum, 0.5" for a disc. Thicknesses beyond twice the above values are not to be recommended.**

For brakes using the high conductivity material the corresponding figures are 0.65" and 1.3" for drum and disc respectively. Again thicknesses beyond twice these values should be avoided.

2.6 Correction for Variable Specific Heat

All the foregoing theory is based on the assumption that the specific heat remains constant at all temperatures within the range concerned, which, as has been already noted, may involve appreciable errors, at least in the case of steel and copper, the specific heat of which varies as shown in Fig.1.

In the case of "thin" discs or drums, i.e. those for which the parameter "a" does not exceed 0.2, a rough correction may be made to allow for variation of specific heat with temperature. For thin discs, the peak temperature is given by eq. (19), and is the ratio of the total heat input to the thermal capacity, if the specific heat is constant.

It seems reasonable to extend the expression, and to say that for a thin disc the peak surface temperature is the ratio of the heat input to the actual thermal capacity; this is certainly true if the disc or drum is infinitely thin, and the only assumption involved is that relating to the maximum thickness to which the expression may be applied. We shall assume that the law is valid up to a = 0.2.

Now the thermal capacity of the disc or drum is

$$DhA \int_0^S c.\,dS = DhA\ I, \text{ say}$$

The integral I is plotted against temperature in Fig.1. The peak temperature is then given by

$$I = \frac{H}{DhA} \tag{20}$$

The procedure is then as follows: calculate I from eq. (20); then read off S from fig.1.

3. THE EFFECT OF COOLING

3.1 Physical Considerations and Assumptions

The cooling problem will be treated in relation to disc brakes only.

Cooling is due to radiation and convection. Convection losses depend on the temperature difference between brake surface and the atmosphere, and on air velocity past the brake disc surface. This velocity will vary from beginning to end of the run. Exact calculation of losses under such conditions could no doubt be carried out by numerical step-by-step methods, but for the purposes of rough estimation it is considered sufficient to regard convection losses as proportional to temperature difference only, the constant of proportionality being based on average conditions through the run. This allows the problem to be treated by ordinary methods of analysis (operational calculus).

Radiation losses are proportional to the difference between the fourth powers of the absolute temperatures of the brake surface and atmosphere. This problem could be dealt with by means of finite difference equations. Before embarking on such an undertaking however it is desirable to determine if radiation effects are an important factor in the total heat loss.

The radiation loss for a steel surface is given by

$$h'' = 0.816\left[\left(\frac{\theta}{1000}\right)^4 - \left(\frac{\theta_o}{1000}\right)^4\right] \text{ cals/sq.cm./sec.}$$

where θ is the absolute temperature of the radiating surface and θ_o the absolute temperature of the surrounding bodies, which will be taken as that of the atmosphere.

The convection loss in calories per square cemtimetre per degree C temperature difference per second is given by

$b = \frac{k'}{d'} Nu$ where k' = conductivity of air
d' = effective hydraulic diameter of air-flow path
Nu = Nusselt's number

Nusselt's number is given by

$$Nu = 0.0277\, R^{0.8}\left(\frac{h'}{d'}\right)^{-0.05}$$

where R = Reynolds' number
h' = length of air flow path

For the typical disc brake referred to in section 2.3, the effective hydraulic diameter $\left(\frac{4 \times \text{area of path}}{\text{perimeter of path}}\right)$ is 0.53" and the length of the path is 2".

We shall take an average air flow velocity of 100 ft/sec. and an average temperature of 200°C, for which the kinematic viscosity is 4×10^{-4} sq.ft./sec., and the conductivity k' is 8×10^{-5} cals/sec/cm/°C.

Thus $$R = \frac{100 \times 0.53 \times 10^4}{12 \times 4} = 11000$$

and hence $$Nu = 0.0277 \times 11000^{0.8} \times \left(\frac{2}{0.53}\right)^{-0.05} = 44.3$$

Therefore $$b = \frac{8 \times 10^{-5} \times 44.3}{0.53 \times 2.54} = 0.00264 \text{ cals/cm}^2/^\circ\text{C/sec.}$$

In the brake in question only 57 per cent of the surface is exposed, the remainder being covered by the brake pads. Hence both convection and radiation loss factors must be multiplied by 0.57 to give effective values, which become:

Convection loss factor b = 0.0015 cals/sq.cm./°C/sec

Radiation loss factor

$$= 0.465\left[\left(\frac{\theta}{1000}\right)^4 - \left(\frac{\theta_o}{1000}\right)^4\right] \text{ cals/sq.cm./sec.}$$

The losses from both sources are plotted against temperature in Fig.5, from which it is seen that even at 600°C (which is on the high side as aircraft brake temperatures go) the convection loss is by far the major factor.

Radiation loss is further reduced (though probably not very substantially) by the fact that much of the disc surface is surrounded by the wheel body, which will eventually warm up and radiate back some of the heat it has received. In multi-disc brakes, radiation losses from all internal disc surfaces must be very small indeed (in some brakes these losses have been increased by interposing a light alloy "heat reservoir" disc between adjoining brake discs).

On the above basis, it would seem that to a fair degree of approximation rediation losses can be neglected in comparison with convection losses. The preceding argument however may not be the whole story, on account of "heat flash" effects, i.e. the development of very high localised transient temperatures. According to Bowden and Tabor (Ref.1), at the very small regions to which contact is localised, temperature almost invariably reaches the melting point of one of the materials of the pair (that which melts first). Parker and Marshall, (Ref.4.), state that experiments on actual brakes do not confirm this, but their results still show transient temperatures considerably in excess of the average.

The effect of flash phenomena would be to enhance radiation losses to a very much greater extent than convection losses, since the former follow a fourth power law, while the latter follow a linear law. On the other hand calculation of additional heat losses due to flash effects hardly seems feasible in the present state of the art. We shall therefore proceed with our calculations on the basis of uniform temperature distribution over the surface, and neglecting radiation effects, but with the mental reservation that these effects may be quite appreciable after all. We shall revert to this point in section 3.3.

3.2 Calculation of Cooling Effects

For the constant torque case, expressions for surface temperature in the presence of convection cooling are developed in Appendix II. It is shown that the temperature at the disc surface is given by

$$S = \frac{2hH}{TkAn}\left\{ \frac{a}{n} \operatorname{erf}\sqrt{t} + 1 - at + \frac{1}{\alpha-\beta}\left[e^{\alpha t}(a-\alpha) + (1-\operatorname{erf}\frac{\alpha}{n}\sqrt{t}) - e^{\beta t}(a-\beta)\left(1+\operatorname{erf} - \frac{\beta}{n}\sqrt{t}\right)\right]\right\} \qquad (21)$$

where

$$\alpha = \frac{n^2}{2} + n\sqrt{1+\frac{n^2}{4}}$$

$$\beta = \frac{n^2}{2} - n\sqrt{1+\frac{n^2}{4}}$$

The constant "n" is a dimensionless cooling parameter given by

$$n = \frac{bh}{k}$$

Thus for the typical brake referred to in sections 2.3 and 3.1, we have h = 0.45" = 1.14 cm., b = 0.0015, k (for steel) = 0.115, and hence n = 0.01485.

For very small values of n eq. (21) involves small differences of large quantities, and becomes quite unmanageable. It is shown in Appendix II that the equation may be approximated by

$$S \approx \frac{2hH}{TkA}\left[\phi_1(t) - nt\left(1 + \frac{t}{2}\right) - a\left[\phi_2(t) - \frac{nt^2}{2}\left(1 + \frac{t}{3}\right)\right]\right] \quad (22)$$

where $\phi_1(t)$ and $\phi_2(t)$ are as defined by equations (8) and (9), or the approximate expressions (10), (11), (12) and (13).

Equation (22) involves appreciable errors at the end of the run unless n is extremely small (say 0.01 or less), but for the purpose of determining the <u>peak</u> temperature only it is probably quite accurate enough up to n = 0.05.

Fig. 3 shows surface temperatures for the brake already referred to, as worked out for n = 0.01, n = 0.1, and n = 0.2. For n = 0.01 eq. (22) has been used, and it is seen that in this case the reduction in temperature due to cooling is quite small, although the assumed cooling coefficient is of the same order as that worked out above. If it is confirmed that this value is roughly representative of current brake practice, it would seem that cooling is a relatively unimportant factor in the performance of present-day brakes.

For large values of t eq. (21) may be written

$$S \approx \frac{2hH}{TkAn}\left[1 + \frac{a}{n} - at - \frac{2}{\alpha - \beta}\; e^{\beta t}(a - \beta)\right] \quad (23)$$

As shown in Appendix VI, under these conditions the peak temperature is given by

$$S_{max} \approx \frac{H}{acDh} \times \frac{2}{\eta}\left[1 + \frac{1}{\eta}\left(1 + \frac{n}{\beta} - \frac{n}{\beta}\log\frac{(\alpha - \beta)}{-2\beta}\right) + \frac{n}{\eta\beta}\log\left(1 - \frac{\beta\eta}{n}\right)\right] \quad (24)$$

where η is the "thin disc" dimensionless cooling parameter defined by

$$\eta = \frac{bT}{cDh} \quad \left(\text{note that } \eta = \frac{n}{a}\right)$$

Eq. (24) is valid for relatively small values of a, since the range of t is from 0 to 1/a; the range of validity of the equation will also depend on n or η, i.e., on the degree of cooling, since the greater the cooling the sooner is the peak temperature reached, i.e., the smaller the value of t at which the peak temperature occurs.

If we write $\alpha \approx -\beta \approx n$, we get

$$S_{max} \approx \frac{H}{AcDh} \times \frac{2}{\eta}\left[1 - \frac{1}{\eta}\log(1 + \eta)\right] \quad (25)$$

an expression which can be derived directly for thin discs (see appendix III).

For the example worked out above, when n = 0.1 (a very high value, as we have seen), eq. (25) gives the peak temperature to within 4 per cent accuracy (in this example, a = 0.0975).

The parameter n is thus no true indication of the degree of cooling, but if the disc is relatively thin a true measure of the degree of cooling is given by the parameter η.

The expression $f(\eta) = \frac{2}{\eta}\left[1 - \frac{1}{\eta}\log(1+\eta)\right]$ measures the ratio of the cooled to the uncooled peak temperatures for a thin disc. $f(\eta)$ is plotted against η in Fig.6.

It is also shown in appendix III that for an infinitely thick disc the ratio of the cooled to the uncooled peak temperature is roughly given by

$$\phi(\lambda) = \frac{1 - e^{\frac{1}{2}\lambda^2}\left(1 - \operatorname{erf}\frac{\lambda}{\sqrt{2}}\right)}{\lambda\sqrt{\frac{2}{\pi}}}$$

where $\lambda = b\sqrt{\frac{2T}{ckD}}$ is the effective cooling parameter for this case.

$\phi(\lambda)$ is plotted against λ in Fig.6; the similarity of the curves of $f(\eta)$ and $\phi(\lambda)$ is striking.

3.3 Experimental Determination of Cooling Coefficients

In view of the very rough assumptions involved in the calculation of the convection cooling coefficient "b" in section 3.1., the typical values quoted must be accepted with some reserve, and it would seem desirable to make direct measurements of the cooling coefficient on actual brakes.

The method proposed here would given an average effective value of an assumedly linear coefficient over a range of temperature from any chosen maximum down to half that maximum; confining measurement to the upper half of the range is sound since errors involved in the lower half would be comparatively insignificant as regards their effect on the final temperature. The coefficient obtained by this method would automatically carry some allowance for the effect of radiation. To derive the full benefit of this, it is desirable to take the initial temperature as that corresponding to the conditions to which the results of the measurements are to be applied.

The proposed method is as follows:

Run the brake under load until the desired peak temperature is reached, and allow to stand until the temperature is roughly equalised through the thickness of the metal. Then run the brake under zero load, allowing it to cool until the temperature has dropped to half the value it had at the beginning of the run, and note the time taken.

Let t_2 be this time, and let $t = \frac{t_2}{t_1} = \frac{kt_2}{cDh^2}$ be the "non-dimensional time".

Then as shown in Appendix IV, the cooling parameter "n" is given by

$$(1 - \frac{n}{2}) \ e^{-n\,(1 - \frac{n}{2})^{t}} = \tfrac{1}{2} \qquad (26)$$

In practice a sufficiently close approximation is obtained from the expression

$$n = \frac{0.693}{t} \qquad (27)$$

or its equivalent

$$\frac{b}{cDh} = \frac{0.693}{t_2} \qquad (28)$$

with an error of 2.5 per cent if n = 0.1 (a very large value). This for likely values of the cooling parameters, equations (27 or (28) are quite good enough.

It would be desirable to carry out measurements at various values of angular velocity up to the maximum for which the brake is intended. With current brake testing facilities it will almost certainly be impossible to reproduce the effect of the forward velocity of the aircraft, but tests reproducing the correct angular velocity should provide at least a rough estimate, the accuracy of which could perhaps be checked by a wind tunnel test for one or two representative cases.

We have stated above that a determination of the cooling coefficient by the proposed method would automatically carry a correction for the effect of radiation, since it involves measurement of the total heat loss. If however, during actual braking radiation losses are enhanced by the flash effects discussed in section 3.1, such effects will not be reproduced under the proposed experimental conditions, and hence, the measured effective cooling coefficient may still be too low.

Additional information could be obtained by comparing calculated and measured temperatures during a braking run, if the former are based on <u>measured</u> cooling coefficients. Since calculation may well involve other errors besides neglect of radiation losses, the comparison can only give rough indications: these may however be sufficient to decide whether or not radiation losses are appreciable.

If it is confirmed that radiation losses - or rather additional radiation losses due to flash effects - are a significant factor, it should be worth while attempting their direct measurement during a breaking run. In the meantime, there does not seem to be sufficient justification to warrant the development of methods of calculation for radiation effects, especially as these methods might have to be based on statistical information as to flash effects, which is not yet available.

4. THERMAL CONDITIONS IN FRICTION MATERIAL

4.1 Reasons for Investigation

The experimental results discussed in section 5 below tend to show that friction lining wear can be greatly influenced by thermal effects other than peak surface temperature. The theory will be considered later that one important factor may be the temperature gradient near the surface of the friction material, and in order to provide material for consideration we shall proceed to calculate the magnitude of this gradient.

4.2 Temperature Gradient Near Surface of Friction Material

Investigation will be confirmed to the case of an uncooled brake, using results already obtained for the surface temperature.

Strictly speaking, of course, the existence of a finite temperature gradient in the friction material implies some flow of heat into the latter. If, however, the amount of heat flowing into the friction material is assumed to be only a very small proportion of the total heat generated, expressions previously derived for surface temperature may be allowed to stand, and the flow of heat into the pad can be calculated from a known surface temperature - time relationship.

As shown in Appendix V and Appendix VI, conditions differ widely according as to whether brakes are applied suddenly or gradually. In the latter case, the temperature gradient near the pad surface grows gradually up to a maximum, after which it decreases. If c_1, k_1, D_1 are respectively the specific heat, conductivity, and density of the friction material, the peak gradient near the surface is roughly given by

$$G_{max} = 0.61 \sqrt{\frac{c_1 D_1}{k_1}} \; R_{max} \, S_{max}$$

where S_{max} is the peak surface temperature, and R_{max} is the peak rate of increase of surface temperature, i.e., the slope of the temperature - time curve at its point of inflection. If is further shown in Appendix VI that the above expression can be written

$$G_{max} = 0.61 \sqrt{\frac{c_1 D_1}{k_1}} \times \frac{H}{AcDh} \times \frac{\Phi(a)}{\sqrt{T}} \qquad (29)$$

where $\Phi(a)$ is plotted against a in Fig.4. For small values of a, $\Phi(a)$ remains practically constant at 1.24. For large values of a the above expression reduces to

$$G_{max} = 1.07 \sqrt{\frac{c_1 D_1}{k_1}} \times \frac{H}{A\sqrt{ckD}} \times \frac{1}{T} \qquad (30)$$

If Φ (a) $\approx$ 1.24, the brake may be said to be of the "thin disc" type as far as temperature gradient is concerned, while if eq. (30) applies, the brake is practically infinitely thick. Here again, as in section 2.5, the limits for the two cases are roughly $a < 0.2$ and $a > 1$.

In the constant torque case it is shown in Appendix V that immediately the brake is applied there is a temperature gradient given by

$$G = \sqrt{\frac{c_1 D_1}{k_1}} \times \frac{4H}{AT} \times \frac{1}{\sqrt{ckD}} \qquad (31)$$

This gradient is obviously independent of thickness and its ratio to the gradient given by equation (29) is $2.64\sqrt{a}$ if a is small and 1.87 if a is large. Unless a is quite small, it seems likely that the maximum gradient occurs at the beginning of the run.

The possible significance of the above findings is descussed in section 5.6 below.

5. SOME EXPERIMENTAL RESULTS AND THEIR INTERPRETATION

5.1 Some Experimental Results

In the course of tests on disc brakes carried out by British Messier Ltd., some significant phenomena became apparent. The full set of results obtained on brakes "D" and "E" mentioned below are given in Fig.9 and Fig.10, but the following deductions are interesting:-

(i) The first phenomenon was that pad wear was considerably reduced if the disc material was changed from steel to a material having a high coefficient of conductivity. Thus the following results were obtained:-

Brake	Wear per stop - ins.	Calculated Peak Temp. - °C.
A - steel discs	0.0115	598
- H.C.M. discs	0.0035	590
B - steel discs	0.00175	662
- H.C.M. discs	0.00068	645
C - steel discs	0.0035	578
- H.C.M. discs	0.0006	573
D - steel discs	0.0092	560
- H.C.M. discs	0.0008	575

Both steel and other discs were chromium plated and hence surface conditions were identical. In spite of this, the H.C.M. discs gave consistently higher coefficients of friction. For brake "D" for instance, the coefficient of friction with the H.C.M. discs was 25 per cent higher than with the steel discs.

(ii) The wear increased appreciably if the chrome plating was omitted. Tests were done with brakes having discs in both conditions and particular results found were as follows.

Brake	Wear per stop - ins.
D - chrome steel discs	0.0092
- plain steel discs	0.0124
E - chrome plated bronze	0.001
- unchromed bronze	0.002

(iii) If the energy and torque were kept constant, an increase in rubbing velocity resulted in an increase in wear. The increase in velocity was obtained by reducing the flywheel inertia on the testing machine. The results shown in Fig.7 were obtained.

(iv) Tests were made on two brakes of similar design but different size, in which the following parameters were very nearly equal for the two tests:

Rubbing Speed

Stopping Time

Energy input per unit volume (and hence, near enough, final mean temperature)

The results were as follows:-

	BRAKE "X"	BRAKE "Y"
Size of discs - in.	11.1 x 6.5 x 0.9	11.9 x 8.6 x 1.5
Energy per unit volume - ft. lb. per cu. in.	26700	29500*
Energy per unit area - ft. lb. per sq. in.	24000	44400
Wear/Stop - in.	0.002	0.010
Pad Pressure - p.s.i.	426	730

* Based on nominal volume corresponding to disc size as given - actual figure is somewhat less due to presence of large lugs on disc.

The difference between the wear figures is striking to say the least, and it is not thought that it is accounted for by the difference in pad pressures alone.

For both brakes "D" and "E" tests were done with varying flywheel inertia but keeping the initial speed and the average torque constant. Thus the following results show the effect of varying the energy input, (Fig.11).

	BRAKE "D"	BRAKE "E"
75 per cent energy	.00006	
100 per cent energy	.00008	.00056
125 per cent energy	.00074	.0028
150 per cent energy	.0018	.0061

5.2 Remarks on Wear

The nature of frictional wear is as yet imperfectly understood. As already mentioned in section 1.3, frictional wear seems to be due to cohesion forces between rubbing surfaces at points of intimate contact, resulting in the shearing off of small particles below the contact surface. According to Ref.1, these cohesion forces are the result of local melting of one of the bodies in contact, but other theories have also been put forward.

Experimental research on the subject to date does not seem to have reached a stage where general conclusions can be formulated as to the effect of all the significant variables. More particularly, most of the fundamental research work has been done with metallic pairs of surfaces, and under conditions widely different from those obtaining in brakes. Much work has, of course, also been done by brake lining manufacturers, but the complexity of the phenomena involved appears to be such that only a few significant factors have been isolated.

For these reasons it can hardly be expected that an investigation such as the present one could provide a clear answer to the problems under discussion. The best that the authors can hope to do is to make a few conjectures on certain aspects, which may be verified by experimental work at a later stage.

5.3 Effect of Bulk Temperature

By "bulk temperature" we shall mean the surface temperature calculated by means of the theory given in preceding sections, on the assumption of a uniform heat input over the surface, or the temperature as measured in the experiments quoted, which was obtained at the end of the run, with sufficient delay to allow irregularities in temperature distribution to disappear through conduction.

The results quoted in section 5.1 show that a substantial decrease in wear can take place while the bulk temperature remains practically constant.

When we consider the comparative tests with steel and H.C.M. discs, both chromium plated, surface conditions, loading and velocity are identical, and hence the differences must lie in thermal effects, even though the peak bulk temperature remains practically unaffected.

If we consider not only the bulk peak surface temperature but the surface temperature throughout the run, there is a marked difference between steel and H.C.M. discs, as shown in Fig.8, as calculated for a typical brake under constant torque conditions. In the early stages, the surface temperature is much lower for the higher conductivity material.

5.4 Deterioration of Friction Material

Heat produces chemical changes in the friction material, particularly above a certain critical temperature, which may be of the order of 200 - 300°C. Thus, if we compare steel and H.C.M. discs, with the former the average temperature through the run is higher and the friction material is exposed to temperatures above the critical value for a longer period of time. One would therefore expect a greater degree of deterioration with steel discs, and consequently a higher rate of wear.

It is, however, difficult to believe that this effect can completely account for the very wide differences in wear observed as between steel and H.C.M. brakes. Besides, when we come to consider the effect of velocity, for a given energy and brake torque (which were the conditions of comparison) the stopping time varies inversely as the velocity, and hence deterioration should decrease with increasing velocity, if the peak temperature remains practically unaltered; thus the effect of deterioration should be, if anything, to reduce the difference between results at high and low velocities.

5.5 Maldistribution on Effects

As already mentioned in section 1.3, two kinds of maldistribution effects may be considered:-

(i) As occurs in rubbing between hard surfaces, when contact is limited to high regions of small extent.

(ii) As some research workers believe is the actual case in brakes, where contact occurs over a comparatively large area though by no means to whole of the pad or shoe.

In case (i) the conductivity of the disc material has a particularly important effect on local "flash" temperatures occurring at points of contact. Furthermore, the points of contact are continually changing, and under such conditions rubbing velocity also has an important effect on "flash" temperatures.

In case (ii) contact tends to be confined to a "band", say an annulus near the centre of the disc in the case of a plate brake, and this annulus tends to get somewhat hotter than the rest of the disc;

effects of this nature are described in Ref.4. Under such conditions it is observed that the disc or drum warps, and such warping naturally has a strong effect on wear. This warping is accounted for by the fact that the thermal stress set up by a relatively hot annulus confined to the centre or edge may easily exceed the elastic limit. The temperature maldistribution in this case is obviously affected by the conductivity of the disc material, and the favourable effect of increasing the latter is therefore explained. The effect of velocity does not seem to be related to this phenomenon.

The explanation just given is valid only if the maldistribution effect is appreciable with steel discs. It certainly appears to have been serious in some early designs of disc brakes, where definite warping was noticed. In more recent designs however, including those quoted in section 5.1, the pad area has been made relatively small, resulting in higher contact pressures and therefore better distribution of load, and no warping has been noticed even with steel discs. Maldistribution effects may of course still be present, but their magnitude must be relatively small, and it would seem rash to conclude that they would entirely account for the beneficial effect of high conductivity.

In case (i) there may be yet another effect of importance. Heat flash effects are confined to a very small volume of metal near the surface, and must therefore be practically unaffected by disc thickness. Hence if these phenomena are a major factor in wear, the effect of disc thickness should be somewhat less than one would otherwise expect. We have shown above (para. 2.5) that for "thin" discs (which include most actual brakes) the peak temperature is proportional to energy per unit disc <u>volume</u>, but we may expect that any effects associated with flashes will be more nearly proportional to energy per unit disc <u>area</u>. Thus if we compare brakes of different sizes, in which energy per unit volume and all other significant parameters are made identical, the thinner brake of the two may have a better performance, since its energy per unit area will be less.

It is interesting to consider brakes "X" and "Y" of section 5.1 in this light. In these two cases it would appear that nearly all significant parameters are very close together for the two cases, except energy per unit area and pad pressure. It is hard to believe however that pad pressure alone would account for a difference of the order of 1 : 5 in the wear, and it seems likely that at least an equally significant difference is that between energies per unit area, which are in the ratio 24 : 44.

Note that the above argument implies that wear is bound to become worse as brake size increases, unless the thickness can be kept constant, which seems impracticable on grounds of size and strength.

In this connection another point is worth bearing in mind. It is generally accepted that single disc designs are preferable to multi-disc types, as the latter lead to maldistribution of load and various other practical difficulties. For a given size however, a multi-disc brake would have a larger area than a single disc type. Hence if it is confirmed that the effect of area is important, the trend towards single disc brakes may not be altogether sound.

5.6 <u>Effect of Temperature Gradient in Friction Material</u>

It has already been noted that according to current theory frictional wear consists in the detachment of small particles from the surface under the action of cohesive forces, which sometimes cause shearing to take place below the contact surface. Such shearing takes place when the ultimate strength of the material is exceeded, and it seems reasonable to suppose that if the material is in a state of initial stress particles will become detached more easily, since a smaller load will be required to reach the ultimate stress. Thus it seems likely that an initial state of stress near the surface of the friction material will accelerate the wearing process.

"Initial state of stress" is used here in the sense of a state of stress additional to that due to normal and friction forces. Such a state would be caused by thermal expansion phenomena if there is a temperature gradient in the material, and would increase with increasing gradient (mention of proportionality is purposely avoided since friction materials do not obey Hooke's law).

Expressions for the temperature gradient near the surface of the friction material have been given in section 4.2, and it will be noted that all these expressions contain some power of the stopping time T in the denominator; thus for a given energy absorbtion the gradient increases with decreasing stopping time, i.e. increases with increasing initial rubbing velocity. Thus, according to the present explanation, the effect of velocity should be to increase wear, which is in accordance with experiment.

As regards the effect of conductivity of disc or drum material, there is a very important difference between conditions of gradual and sudden application of brakes. With the former, the peak temperature gradient is given by eq. (29); now for most practical brakes (at least those of the disc type) the parameter "a" seldom exceeds 0.2 for steel, and hence $\Phi(a)$ shows very little excess over its minimum value of 1.24; thus a reduction in "a", which would be brought about by an increase in conductivity, will have practically no effect on the peak temperature gradient.

Thus if temperature gradient is the major factor in the effect of conductivity on wear, it is to be expected that for most practical brakes an increase of conductivity will have very little effect in the case of gradual application, if the torque - time characteristics are anywhere near those assumed in section 2.2. Experiments could easily be made to verify this prediction, by testing brakes under gradually applied load conditions.

Under constant torque, or more generally under suddenly applied torque conditions, the picture is entirely different. There is a high initial temperature gradient given by eq. (31), and the magnitude of this gradient varies inversely as the square root of the conductivity of the disc or drum material. This initial gradient may or may not be exceeded at some later stage in the run (depending on the stopping time and the proportions of the brake), but even if it is exceeded at some stage, it may still be the most important factor for the following reasons:-

(i) At the beginning of the run velocity is high, and velocity may have an important effect on wear apart from thermal gradient considerations.

(ii) At the beginning of the run temperature is low, and it may well be that the combination of low temperature with high temperature gradient is particularly harmful, since when the friction material is cool its rigidity is a maximum, and hence a given temperature gradient will cause the maximum stress.

It seems not unlikely therefore that under suddenly applied torque conditions, an increase of conductivity decreases wear because it reduces the initial temperature gradient in the lining material.

5.7 Effect of Brake Design

Brake design has already been implicitly discussed in some of the foregoing matter, and in this section only one particular problem will be examined.

In the early disc brakes, the pads nearly covered the whole area of the disc. It has been the experience of several brake manufacturers that much better results (particularly from the point of view of friction pad wear) are obtained with a design in which there are two segmental pads at diametrically opposite stations on the disc, and the pad carrier is cut away to leave the remainder of the disc exposed.

There seems to be an impression that the improvement is due to better cooling conditions, resulting from exposure of a large portion of the disc. In the authors' opinion this explanation is not tenable. Reducing the pad area does certainly increase the amount of disc surface subjects to cooling, although if cooling effects are as small as calculated the nett effect can hardly be very important. Cutting away the pad carrier however should have very little additional effect on radiation, since any portion of the disc not covered by pads will radiate, whether fully "exposed" or not; the only difference would in the amount of heat radiated back by the pad carrier, which can hardly be very large. As regards convection, it would seem that the only favourable effect of the cutaway could be to increase the speed of air flow over the disc. We have shown however that even under favourable assumptions as to this speed, convection cooling should be quite small, and the improvement obtainable from the cutaway must be smaller still.

Thus the modern "segmental" as opposed to the old-fashioned "annular" type of disc brake may be better in respect of cooling, but only because the disc surface is not covered by pads to the same extent, and not because the pad carrier is cut away "to expose the disc". Since, however, cooling does not seem to be a major factor in any event, further explanations must be sought to account for the improvement in performance. Possible explanations are:-

(i) Probably the most important, i.e. the direct effect of reduction in pad area, resulting in increase of pad pressure. As mentioned in section 1.3, up to a certain point an increase in pad pressure results in a more uniform distribution of normal load over the area of the pad, which has a beneficial effect on wear. This has been confirmed experimentally in the case of railway brakes (Ref.4). There is an optimum value of pad pressure for which wear will be a minimum. Unfortunately this value will vary for different materials, and will probably also vary with peak disc temperature and possibly other parameters as well. Hence no recommendation can as yet be made as to the optimum value of pressure.

Yet another factor may be of significance, far-fetched as it may sound.

(ii) Friction is a complex physicochemical phenomenon, in which the chemical condition of the rubbing surfaces plays a very important part. The coefficient of friction and the wear are particularly high for deoxidised and chemically clean surfaces. It seems not impossible that although no measurable wear of the disc may occur, there is still a kind of wear in the sense of removal of a surface oxide film, which builds up again at a very rapid rate as the affected portion of the surface leaves the pad and becomes exposed to the atmosphere. If such an effect exists, the amount of repair to the oxide film would increase as the gaps between the pads increased.

6. CONCLUSIONS

6.1 Temperature Calculation

(i) The expressions given in section 2.4 enable peak surface temperatures to be rapidly determined if the effect of cooling is neglected. For an efficient brake it seems desirable to

exceed the value of 0.25" thickness (drum) or 0.5" (disc) as little as possible; in any event an increase beyond twice the above values has very little effect on temperature. The above figures apply to steel, and the corresponding figures for H.C.M. are 0.65" (drum) and 1.3" (disc). The above figures apply for a 16 second run; for other values of stopping time, the figures vary as the square root of the latter.

(ii) The effect of cooling can be calculated with the use of rather rough assumptions and in certain cases a rapid estimate is possible. From calculated convection coefficients this effect seems to be very small in representative modern brakes, but it would be desirable to obtain experimental values for the parameters involved; methods of doing this have been given in section 3.3.

6.2 <u>Effect of Conductivity of Disc or Drum Material</u>

(i) The beneficial effect on wear of increased conductivity is not due to lowering of the peak surface temperature, at least for the experimental results quoted.

(ii) Increased thermal conductivity should have some favourable effect on thermal deterioration of the friction material, but this is not believed to be an important factor in the very marked improvement observed in tests.

(iii) Increased thermal conductivity may have a favourable effect by reducing the effect of maldistribution of heat input due to localisation of contact between friction material and disc or drum. There seems to be differences of opinion as to whether contact is localised to a number of small spots, or to an annular band; if the latter, it appears that the effect of localisation is not always a serious one, and hence the explanation may not apply to all cases.

(iv) Increased conductivity may act by reducing temperature gradients in the friction material. In practice, this effect should be important only if the brake load is applied suddenly, and only if a substantial amount of wear takes place during early stages in the run. Experiments are desirable to verify both the above conclusions.

6.3 <u>Effect of Rubbing Velocity</u>

(i) The unfavourable effect on wear of high initial rubbing velocities does not appear to be wholly accounted for by an increase in peak surface temperatures, since for the experimental results quoted this increase was quite small.

(ii) The effect in question may be due in part at least to localisation of contact to a number of small areas, if such localisation does indeed occur. Under such conditions, velocity would have an important effect on the very high "flash" temperatures occurring at points of contact.

(iii) The unfavourable effect of velocity may also be due (at least in part) to the fact that an increase in rubbing speed would result in an increase in temperature gradient in the friction material, for a given energy input and brake torque.

(iv) The present state of knowledge as regards wear is inadequate to obtain a really complete appraisal of the problem at hand.

(v) No special methods can be suggested for mitigating the effect of velocity.

6.4 Possible Methods of Reducing Wear

(i) Increasing Thickness of Disc or Drum - this reduces peak surface temperature, but only within limits; under suddenly applied load conditions, if it is confirmed that substantial wear takes place in the early stages of the run, the effect of thickness may be quite small.

(ii) Increasing Conductivity of Disc or Drum Material - this needs no further discussion, but it must be pointed out once more that if brake load is applied gradually, it may turn out that the effect is quite small.

(iii) Increasing Specific Heat of Disc or Drum Material - this should be beneficial under all conditions but no suitable materials seem to be available at present (as a matter of interest, pure beryllium has high specific heat, heat conductivity and low density).

(iv) Ensuring Gradual Application of Brake Torque, subject to experimental verification of the effect of rate of application.

(v) Use of High Conductivity Friction Material - this may help if it is confirmed that temperature gradients in the friction material are a major factor. It is understood however that high conductivity friction materials are viewed with some suspicion, due to the fear that thermal deterioration may spread too rapidly through the thickness of the material.

(vi) Increased Cooling - the degree of cooling achieved at present seems to be so low that an increase to really effective values may be impracticable.

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APPENDIX I

Surface Temperature Determination by Existing Methods (Uncooled Brakes)

It is shown in Ref. 2 that for any arbitrary rate of heat input specified as a function of time ($Q = Q(t_2)$) the surface temperature is given by

for $t < 1/3$

$$S = \frac{h}{kA\sqrt{3}} \int_0^t \frac{Q(u)du}{\sqrt{t-u}}$$

for $t > 1/3$

$$S = \frac{h}{kA} \left[\int_0^{t-1/3} Q(u)du + \frac{1}{\sqrt{3}} \int_{t-1/3}^{t} \frac{Q(u)du}{\sqrt{t-u}} \right]$$

Under constant torque conditions the deceleration is constant, and hence the velocity decreases linearly with time, and so does the rate of heat input. Hence

$$Q = \frac{2H}{T} \left(1 - \frac{t_2}{T}\right) = \frac{2H}{T}(1 - at)$$

i.e. $$Q(u) = \frac{2H}{T}(1 - au)$$

Substituting in the above expressions and performing the integrations, we obtain Eq's. (5) and (6).

APPENDIX II

General Theory, and Alternative Method of Calculating Surface Temperatures

It will be assumed that at any given instant the rate of heat input per unit area is constant in the axial direction for a disc brake, and in the radial direction for a drum brake. Since brake pads or shoes do not cover the whole area of the disc or drum, instantaneously the rate of heat input is not constant in a circumferential direction. In practice however the rate of rotation is such as to allow the assumption that the effective heat input at any point is the average for the whole brake. In other works, circumferential temperature gradients can be neglected for present purposes, without appreciable error except very near the end of the run, when the rate of heat input is very small in any event.

We can then use the well known conduction equation for a slab, viz.

$$\frac{\partial^2 S}{\partial x^2} = \frac{Dc}{K}\frac{\partial S}{\partial t_2} \tag{32}$$

where x is the co-ordinate perpendicular to the heated surface.

For a slab of finite thickness it is convenient to introduce the non-dimensional co-ordinates

$$X = \frac{x}{h} \text{ and } t = \frac{t_2}{t_1} = \frac{kt_2}{cDh^2}$$

giving

$$\frac{\partial^2 S}{\partial X^2} = \frac{\partial S}{\partial t} \tag{33}$$

The positive direction of x will be taken <u>outward</u> from the heated surface, and the origin at the opposite surface for a drum, or the central plane for a disc; thus at the heated surface $X = 1$. In the case of the drum, we shall assume that there is no loss of heat from the outside surface, and hence, here again, there is no heat flow at $x = 0$. Thus we get the boundary condition for finite slabs

$$\frac{\partial S}{\partial X} = 0 \quad \text{at} \quad X = 0 \tag{34}$$

Let the average loss per unit area be bS, it being assumed that the loss is confined to the rubbing surface. Then at the heated surface

$$k\frac{\partial S}{\partial x} = \frac{Q}{A} - bS \qquad (x = h) \tag{35}$$

or

$$\frac{\partial S}{\partial X} = \frac{hQ}{kA} - \frac{hbS}{k} \qquad (X = 1) \tag{36}$$

Putting $\qquad \frac{hb}{k} = n$

$$\frac{\partial S}{\partial X} = \frac{hQ}{kA} - nS \qquad (X = 1) \tag{37}$$

We multiply eq. (33) by e^{-pt} and integrate from 0 to ∞. The temperature $S(X,t)$ is then replaced by its Laplace transform

$$\bar{S}\,(X,p) = \int_0^\infty e^{-pt}\, S(X,t)dt$$

and the heat supply by its Laplace transform

$$\bar{Q}(p) = \int_0^\infty e^{-pt}\, Q(t)dt$$

Eq.(33) then transforms to

$$\frac{d^2\bar{S}}{dX^2} = p\bar{S} - S_o \tag{38}$$

where S_o is the initial value of S, which will be taken as zero except when we come to consider the case of the cooling of an initially heated brake.

The boundary conditions (34) and (37) become respectively

$$\frac{d\bar{S}}{dX} = 0 \qquad (X = 0) \tag{39}$$

and

$$\frac{d\bar{S}}{dX} + n\bar{S} = \frac{h\bar{Q}}{kA} \qquad (X = 1) \tag{40}$$

The solution of eq.(38) with $S_o = 0$ is

$$\bar{S} = C \cosh X\sqrt{p} + B \sinh X\sqrt{p}$$

From (39), $B = 0$

$$\text{From (40), } C = \frac{h\bar{Q}}{kA} \times \frac{1}{\sqrt{p}\,\sinh\sqrt{p} + n\cosh\sqrt{p}} \tag{41}$$

As we have seen in Appendix I, for the constant torque case

$$Q = \frac{2H}{Tp}\left(1 - \frac{a}{p}\right) \tag{42}$$

Inserting in equation (41), we get

$$\frac{kAT\bar{S}}{2hH} = \frac{1}{p}\left(1 - \frac{a}{p}\right)\frac{\cosh X\sqrt{p}}{\sqrt{p}\,\sinh\sqrt{p} + n\cosh\sqrt{p}}$$

At the surface $(X = 1)$, let $\bar{S} = \bar{S}_1$. Then

$$\frac{kAT\bar{S}1}{2hH} = \frac{1}{p}\left(1 - \frac{a}{p}\right) \frac{1}{n + \sqrt{p}\tanh\sqrt{p}} \tag{43}$$

The interpretation of this transform is extemely difficult and the problem will be solved by substituting an approximation for the term $\sqrt{p}\tanh\sqrt{p}$, viz. $p(1 + p)^{-\frac{1}{2}}$. This approximation is good for small values of $u = \sqrt{p}$, since

$$u \tanh u = u^2 - u^4 + O(u^6)$$

$$u^2(1 + u^2)^{-\frac{1}{2}} = u^2 - \tfrac{1}{2}u^4 + O(u^6)$$

The approximation is also good for large values of u, since

$$u \tanh u = u - 2ue^{-2u} + O(ue^{-4u})$$

$$u^2(1 + u^2)^{-\frac{1}{2}} = u - \frac{1}{2u} + O(u^{-3})$$

The approximation is reasonably good in the whole range $0 < u < \infty$, as seen from the following table

u	$u \tanh u$	$u^2(1 + u^2)^{-\frac{1}{2}}$
0	0	0
0.5	0.231	0.233
1.0	0.7616	0.707
2.0	1.928	1.785
3.0	2.985	2.84
5.0	4.995	4.9
10.0	10.0	9.95
15.0	15.0	14.96
20.0	20.0	19.96

Thus $$\frac{1}{n + \sqrt{p}\tanh\sqrt{p}} \approx \frac{1}{n + p(1 + p)^{-\frac{1}{2}}} = \frac{n(1 + p) - p(1 + p)^{-\frac{1}{2}}}{n^2(1 + p) - p^2}$$

$$= \frac{p(1 + p)^{\frac{1}{2}} - n(1 + p)}{(p - \alpha)\ (p - \beta)}$$

where α, β are the roots of $p^2 - n^2(1 + p) = 0$

Say

$$\alpha = \frac{n^2}{2} + n\sqrt{1 + \frac{n^2}{4}}$$

$$\beta = \frac{n^2}{2} - n\sqrt{1 + \frac{n^2}{4}}$$

When there is no cooling, n = 0, and hence

$$\frac{kAT\bar{S}_1}{2hH} = \frac{1}{p}\left(1 - \frac{a}{p}\right)\frac{1}{\sqrt{p}\,\tanh\sqrt{p}} \approx \frac{1}{p}\left(1 - \frac{a}{p}\right)\frac{(1+p)^{\frac{1}{2}}}{p}$$

$$= \frac{(1+p)^{-\frac{1}{2}}}{p} + \frac{(1+p)^{-\frac{1}{2}}}{p^2} - \frac{a(1+p)^{-\frac{1}{2}}}{p^2} - \frac{a(1+p)^{-\frac{1}{2}}}{p^3}$$

The interpretation of these transforms is worked out in Appendix VII. Using results obtained there, we get

$$\frac{kATS_1}{2hH} = \phi_1(t) - a\,\phi_2(t)$$

where $\phi_1(t) = \frac{1}{2}(t + \frac{1}{2})\,\mathrm{erf}\sqrt{t} + \frac{1}{\sqrt{\pi}}\,e^{-t}\sqrt{t}$

$$\phi_2(t) = \frac{1}{2}\left[t\,(t+1) - \frac{1}{4}\right]\mathrm{erf}\sqrt{t} + (t + \tfrac{1}{2})\,\frac{\sqrt{t}}{2\sqrt{\pi}}e^{-t}$$

When t is small write

$$\mathrm{erf}\sqrt{t} \approx \frac{2}{\sqrt{\pi}}\sqrt{t}\left(1 - \frac{t}{3} + \frac{t^2}{10} - \frac{t^3}{42}\right)$$

and $e^{-t} \approx 1 - t + \frac{t^2}{2} - \frac{t^3}{6}$

Then $\phi_1(t) \approx 2\sqrt{\frac{t}{\pi}}\left(1 + \frac{t}{3} - \frac{t^2}{30}\right) \approx 2\sqrt{\frac{t}{\pi}}\left(1 + \frac{t}{3}\right)$

with an error of probably less than $\frac{t^2}{30}$

$$\phi_2(t) \approx \frac{4}{3\sqrt{\pi}}t^{\frac{1}{2}}\left(1 + \frac{t}{5} - \frac{327}{820}t^2\right) \approx \frac{4t^{3/2}}{3\sqrt{\pi}}\left(1 + \frac{t}{5}\right)$$

with an error of probably less than $\frac{327}{820}t^2$.

When t is large, write $\mathrm{erf}\sqrt{t} \approx 1 - \frac{e^{-t}}{\sqrt{\pi t}}$

Then $\phi_1(t) \approx t + \frac{1}{2} - \frac{e^{-t}}{2\sqrt{\pi t}} \approx t + \frac{1}{2}$

with an error of the order of $\frac{e^{-t}}{2\sqrt{\pi t}}$

$$\phi_2(t) \approx \tfrac{1}{2}\left[t\,(t+1) - \tfrac{1}{4}\right] - \frac{e^{-t}\sqrt{t}}{2\sqrt{\pi}}\left(1 - \frac{1}{4t}\right)$$

$$\approx \tfrac{1}{2}\left[t\,(t+1) - \tfrac{1}{4}\right]$$

with an error of the order of $\dfrac{e^{-t}}{2}\sqrt{\dfrac{t}{\pi}}\left(1 - \dfrac{1}{4t}\right)$

Values of the functions and their approximations work out as follows:-

	$t =$	0.5	1	2
$\phi_1(t)$	Exact Value	0.925	1.468	2.943
	$2\sqrt{\frac{t}{\pi}}\left(1 + \frac{t}{3}\right)$	0.93	1.505	2.66
	$t + \frac{1}{2}$	1	1.5	2.5
$\phi_2(t)$	Exact Value	0.292	0.871	2.875
	$\frac{4}{3}\frac{t^{\frac{1}{2}}}{\sqrt{\pi}}\left(1 + \frac{t}{5}\right)$	0.292	0.9	2.98
	$\frac{t}{2}(t+1) - \frac{1}{8}$	0.25	0.87	2.875

Thus, with an error of not more than about 3 per cent, the approximations for small t's are valid for $0 < t < 1$, and the approximations for large t's are valid for $t > 1$.

When cooling is taken into account $(n \neq 0)$, we write

$$\frac{kAT\bar{S}_1}{2hH} = \frac{1}{p}\left(1 - \frac{a}{p}\right)\left(\frac{p(1+p)}{(p-\alpha)\,(p-\beta)\,(1+p)^{\frac{1}{2}}} - \frac{n(1+p)}{(p-\alpha)\,(p-\beta)}\right)$$

$$= \frac{(p-a)\,(1+p)}{p(p-\alpha)\,(p-\beta)\,(1+p)^{\frac{1}{2}}} + \frac{n(1+p)\,(a-p)}{p^2\,(p-\alpha)\,(p-\beta)}$$

$$= \frac{A_1}{p(1+p)^{\frac{1}{2}}} + \frac{B_1}{(p-\alpha)\,(1+p)^{\frac{1}{2}}} + \frac{C_1}{(p-\beta)\,(1+p)^{\frac{1}{2}}}$$

$$+ \frac{A_2}{p} + \frac{B_2}{p^2} + \frac{C_2}{p-\alpha} + \frac{D_2}{p-\beta}$$

where A, B, C, D are to be found by the method of partial fractions.

The interpretation of these transforms is worked out in Appendix VII. Using results obtained there, we get

$$\frac{TkAS_1}{2hH} = A_1\operatorname{erf}\sqrt{t} + \frac{B_1 e^{\alpha t}\operatorname{erf}\sqrt{(1+\alpha)t}}{(1+\alpha)^{\frac{1}{2}}} + \frac{C_1 e^{\beta t}\operatorname{erf}\sqrt{(1+\beta)t}}{(1+\beta)^{\frac{1}{2}}}$$

$$+ A_2 + B_2 t + C_2 e^{\alpha t} + D_2 e^{\beta t} \tag{44}$$

Recalling that $(p - \ )(p - \ ) = p^2 - n^2(1 + p)$, and that therefore

$$(1+\alpha)^{\frac{1}{2}} = \frac{\alpha}{n}, \qquad (1+\beta)^{\frac{1}{2}} = \frac{-\beta}{n}$$

the coefficients can be determined by the usual methods and work out as follows:-

$$A_1 = \frac{a}{n^2}, \quad \frac{B_1}{(1+\alpha)^{\frac{1}{2}}} = -C_2 = \frac{-(a-\alpha)}{n(\alpha-\beta)}$$

$$A_2 = \frac{1}{n}, \quad \frac{C_1}{(1+\beta)^{\frac{1}{2}}} = D_2 = \frac{-(a-\beta)}{n(\alpha-\beta)}, \quad B_2 = \frac{-a}{n}$$

Substituting these values into eq. (44), we get

$$\frac{TkAS_1}{2hH} = \frac{1}{n}\left[\frac{a}{n}\operatorname{erf}\sqrt{t} + 1 - at + \frac{1}{\alpha-\beta}\left[e^{\alpha t}(a-\alpha)\left(1 - \operatorname{erf}\frac{\alpha}{n}\sqrt{t}\right)\right.\right.$$

$$\left.\left.-e^{\beta t}(a-\beta)\left\{1 + \operatorname{erf}\left(-\frac{\beta}{n}\sqrt{t}\right)\right\}\right]\right] \tag{45}$$

Equation (45) becomes unmanageable when n is very small, say of the order of 0.01. To obtain a solution for such cases, we write

$$\frac{TkAS_1}{2hH} = \frac{1}{n}\left[1 - \frac{1}{\alpha-\beta}\left(\alpha e^{\alpha t}\left(1-\operatorname{erf}\sqrt{(1+\alpha)\,t}\right)\right.\right.$$

$$\left. - \beta e^{\beta t}\left(1 + \operatorname{erf}\sqrt{(1+\beta)\,t}\right)\right) + a\left(\frac{1}{n}\operatorname{erf}\sqrt{t} - t\right)$$

$$-\frac{1}{\alpha-\beta}\left[\left(e^{\alpha t}\left(1-\operatorname{erf}\sqrt{(1+\alpha)\,t}\right)\right)\right.$$

$$\left.\left. - e^{\beta t}\left(1 + \operatorname{erf}\sqrt{(1+\beta)\,t}\right)\right]\right] \tag{46}$$

We can regard erf $\sqrt{(1+\alpha)t}$ as a function of α, and expand by Maclaurin's theorem. If we stop at terms of α^4, this gives

$$\operatorname{erf}\sqrt{(1+\alpha)t} = \operatorname{erf}\sqrt{t} - e^{-t}\sqrt{\frac{t}{\pi}}\left[-\alpha + \frac{\alpha^2}{2}\left(t+\tfrac{1}{2}\right) - \frac{\alpha^3}{6}\left(t^2+t+\tfrac{3}{4}\right)\right.$$

$$\left. + \frac{\alpha^4}{24}\left(t^3 + \frac{3}{2}t^2 + \frac{9}{4}t + \frac{15}{8}\right)\right]$$

Similarly we may expand $e^{\alpha t}$ in the usual manner. Substituting in (46), we get an expression with factors of the type

$$\frac{\alpha^m + \beta^m}{\alpha - \beta} \text{ and } \frac{\alpha^m - \beta^m}{\alpha - \beta}. \quad \text{Now } \frac{1}{\alpha - \beta} = \frac{1}{2n\left(1+\frac{n^2}{4}\right)^{\frac{1}{2}}} \approx \frac{1}{2n}\left(1 - \frac{n^2}{8} + \frac{3n^4}{128}\right)$$

while $\alpha + \beta = n^2$, $\alpha\beta = -n^2$.

Hence the final expression can be written as a series in ascending powers of n; if we take terms as far as the first power only, we get

$$\frac{TkAS_1}{2hH} = \phi_1(t) - nt\left(1 + \frac{t}{2}\right) - a\left[\phi_2(t) - \frac{nt^2}{2}\left(1+\frac{t}{3}\right)\right] \qquad (47)$$

For the example worked out in section 3.2, the curves for which are given in Fig.3, for $n = 0.1$ the above expression gives an error of -9 per cent in the peak temperature (the error is considerably larger at the end of the run). We may conjecture that for $n < 0.05$ equation (22) will be quite accurate enough as a means of determining the peak temperature. For $n = 0.05$ the error is likely to be less than 5 per cent, perhaps considerably less, since there are grounds for believing that the error varies as n^2.

In subsequent problems we shall have occasion to consider the case of a slab of infinite thickness. In this case, it is more convenient to take the origin at the rubbing surface, and to take the positive direction of the coordinate x as being into the slab. Dimensionless coordinates X and t are now devoid of significance.

The basic equation (32) remains unchanged, but for its transformed version (corresponding to (38)) we must write

$$\frac{d^2\bar{S}}{dx^2} = \frac{Dc}{k}(p\bar{S} - S_o) \qquad (48)$$

The boundary condition (34) disappears, and is replaced by the condition that S must remain finite as x approaches infinity. The boundary condition at the heated surface (40) is replaced by

$$\frac{d\bar{S}}{dx} - \frac{b\bar{S}}{k} = -\frac{\bar{Q}}{kA} \quad (x = 0) \qquad (49)$$

Since S must remain finite for all values of x, the solution of equation (48) is (with $S_o = 0$)

$$\bar{S} = Ze^{-x\sqrt{\frac{Dc}{k}}\sqrt{p}}$$

Inserting the boundary condition (47)

$$\left(\sqrt{p}\sqrt{\frac{Dc}{k}} + \frac{b}{k}\right) Z = \frac{\bar{Q}}{kA}$$

Hence

$$\bar{S} = \frac{\bar{Q}}{A\sqrt{cDk}\left(\frac{b}{\sqrt{cDk}} + \sqrt{p}\right)} e^{-x\sqrt{\frac{Dc}{k}}\sqrt{p}}$$

i.e.

$$\bar{S} = \frac{\bar{Q}}{A\sqrt{cDk}\,(m + \sqrt{p})} e^{-x\sqrt{\frac{Dc}{k}}\sqrt{p}} \tag{50}$$

where $\quad m = \frac{b}{\sqrt{cDk}}$

APPENDIX III

Effect of Cooling for "Thin" and "Thick" Discs

For a thin disc we assume that the temperature is uniform throughout the thickness. The basic equation is then

$$c\,D\,h\,A\,\frac{dS}{dt_2} = \frac{2H}{T}\left(1 - \frac{t_2}{T}\right) - bSa$$

i.e., $$\frac{dS}{dt_2} + \frac{bS}{cDh} = \frac{2H}{cDhAT}\left(1 - \frac{t_2}{T}\right) = \frac{dS}{dt_2} + rS \qquad (51)$$

where $$r = \frac{b}{cDh}$$

With $S = 0$ when $t_2 = 0$, the solution of the differential equation is

$$S = \frac{2H}{rcDhAT}\left[\left(1 + \frac{1}{rT}\right)(1 - e^{-rt_2}) - \frac{t_2}{T}\right]$$

The peak temperature occurs when $\frac{dS}{dt_2} = 0$, i.e., when

$$1 - e^{-rt_2} = \frac{rT}{1 + rT}$$

and therefore

$$rt_2 = \log(1 + rT) = \log(1 + \eta)$$

where $$\eta = rT = \frac{bT}{cDh}$$

If $S' = \frac{H}{AcDh}$ is the peak temperature in the uncooled case, we get

$$\frac{S_{max}}{S'} = \frac{2}{\eta}\left[1 - \frac{1}{\eta}\log(1 + \eta)\right] = f(\eta)$$

$f(\eta)$ is plotted against η in Fig.6., and represents the ratio of the peak temperatures in the cooled and uncooled cases, for thin discs.

Let us now consider a disc of infinite thickness. In this case we must make use of eq.(48), viz.,

$$\bar{S} = \frac{\bar{Q}}{A\sqrt{cDk}\,(m + \sqrt{p})}\, e^{-x\sqrt{\frac{cD}{k}}\sqrt{p}}$$

For the constant torque case the solution turns out to be too cumbersome to lend itself to analysis. We shall therefore consider the case of constant heat input, i.e., $Q = Q_o$ = constant. Although

this condition is highly artificial as applied to brakes, it is thought that the general laws as to the effect of cooling can still be ascertained by such means.

With $Q = Q_o$, we get

$$\bar{Q} = \int_o^\infty e^{-pt_2} Q_o dt_2 = \frac{Q_o}{p}$$

Hence $$\bar{S} = \frac{Q_o}{A\sqrt{cDk}\,(m + \sqrt{p})p}\, e^{-x\sqrt{\frac{cD}{k}}\sqrt{p}}$$

At the surface, when $x = 0$

$$\bar{S}_1 = \frac{Q_o}{A\sqrt{cDk}\; p(m + \sqrt{p})}$$

From tables of Laplace transforms, the solution for this is

$$\bar{S} = \frac{Q_o}{mA\sqrt{cDk}} \left[1 - e^{m^2 t_2}(1 - \mathrm{erf}\; m\sqrt{t_2}) \right]$$

In the uncooled case the solution is

$$S = \frac{Q_o}{A\sqrt{cDk}} \frac{2}{\sqrt{\pi}} \sqrt{t_2}$$

In both cases the peak temperature occurs at the end of the run, when $t_2 = T$. Put

$$m\sqrt{T} = b\sqrt{\frac{T}{cDk}} = \frac{\lambda}{\sqrt{2}}$$

In the uncooled case the peak temperature is

$$S' = \frac{Q_o}{A\sqrt{cDk}} \frac{2}{\sqrt{\pi}} \sqrt{T}$$

In the cooled case the peak temperature is

$$S_{max} = \frac{Q_o}{mA\sqrt{cDk}} \left[1 - e^{\frac{\lambda^2}{2}} (1 - \mathrm{erf}\, \frac{\lambda}{\sqrt{2}}) \right]$$

Thus $$\frac{S_{max}}{S'} = \frac{1 - e^{\frac{\lambda^2}{2}} (1 - \mathrm{erf}\, \frac{\lambda}{\sqrt{2}})}{\lambda\sqrt{\frac{2}{\pi}}} = \phi(\lambda)$$

$\phi(\lambda)$ represents the ratio of the peak temperatures in the cooled and uncooled cases, and is plotted against λ in Fig.6.

APPENDIX IV

Determination of Cooling Parameters by Temperature Measurements

Let the brake be heated and then brought to a stop. Assume that when stopped the rate or cooling is relatively low, and that therefore the temperature will equalise through the thickness of the disc. Now let the brake be rapidly spun up to the required speed, which will be that for which the cooling parameters are to be determined. Let the brake be kept spinning without any heat input, and let us calculate the drop in surface temperature under such conditions.

Let S_o be the initial temperature. The basic equation (38) still applies, as does the boundary condition (39). The second boundary condition (40) must be modified to

$$\frac{d\bar{S}}{dX} + n\bar{S} = 0 \quad (x = 1) \tag{51}$$

Solving (38) and (39) and putting x = 1, we get

$$\bar{S}_1 = \frac{S_o}{p} + C \cosh\sqrt{p}$$

Eq. (51) gives

$$\frac{nS_o}{p} + \sqrt{p}\, C \sinh\sqrt{p} + n\, C \cosh\sqrt{p} = 0$$

Hence the solution is

$$\bar{S}_1 = \frac{S_o}{p}\left(1 - \frac{n}{p(n + \sqrt{p}\tanh\sqrt{p})}\right)$$

This is solved in the same manner as analogous equations in Appendix II, by putting $\sqrt{p}\tanh\sqrt{p} \approx p^{\frac{1}{2}}(1 + p)^{-\frac{1}{2}}$, the solution being

$$S = \frac{S_o}{\alpha - \beta}\left[\alpha e^{\alpha t}\left(1 - \mathrm{erf}\,\alpha\frac{\sqrt{t}}{n}\right)\right.$$

$$\left. - \beta e^{\beta t}\left(1 + \mathrm{erf} - \beta\frac{\sqrt{t}}{n}\right)\right] \tag{53}$$

where α, β have the values defined in Appendix II.

Equation (53) is the solution for the general case. As a matter of interest, we shall also obtain solutions for the limiting cases of "thin" and "thick" discs.

For an infinitely thick disc we take equation (48), the solution of which is

$$\bar{S} = Z\, e^{-x\sqrt{\frac{cD}{k}}\sqrt{p}} + \frac{S_o}{p}$$

The boundary condition (43) becomes now

$$\frac{d\bar{S}}{dx} - \frac{b\bar{S}}{k} = 0 \quad (x = 0) \tag{54}$$

which gives

$$Z\left(\sqrt{\frac{cD}{k}}\sqrt{p} + \frac{b}{K}\right) = \frac{-bS_o}{kp}$$

Solving this and putting $x = 0$, we get the following expression for the surface temperature

$$S = S_o e^{m^2 t_2}\,(1 - \mathrm{erf}\; m\sqrt{t_2}) \tag{55}$$

where m is as defined in Appendix III.

For a thin disc we use eq.(51) with $H = 0$, i.e.,

$$\frac{dS}{dt_2} + rS = 0$$

with the boundary condition $S = S_o$ when $t_2 = 0$. The solution is then

$$S = S_o e^{-rt_2} \tag{56}$$

Let us now revert to the general case, as expressed by eq.(53). Let the test be made to measure the time taken for the temperature to drop to half its initial value. Even when n is as high as 0.1, the value of t under such conditions is so large that the first term in eq.(53) becomes negligible, and we can write

$$\frac{S}{S_o} = -\frac{2\beta}{\alpha - \beta}\, e^{\beta t}$$

Again we can write

$$\beta \approx -n\left(1 - \frac{n}{2}\right), \quad \alpha - \beta \approx 2n$$

and thus

$$\frac{S}{S_o} \approx \left(1 - \frac{n}{2}\right) e^{-n(1 - \frac{n}{2})t} \tag{57}$$

The above expression is still accurate enough for

$$\frac{S}{S_o} = \frac{3}{4}$$

For $\frac{S}{S_o} = \frac{1}{2}$, we get

$$\frac{1}{2} = \left(1 - \frac{n}{2}\right) e^{-n(1 - \frac{n}{2})t}$$

With n = 0.1, this gives t = 6.75.

If we use the "thin disc" expression eq.(56), for $\frac{S}{S_o} = \frac{1}{2}$ we must have $rt_2 = \log 2 = 0.693$. Now $rt_2 = nt$, and hence we should get $n = \frac{0.693}{t}$. Thus if t = 6.75, this would give n = 0.1025. Thus when n = 0.1 the expression $n = \frac{0.693}{t}$ or its equivalent $r = \frac{0.693}{t}$ give the value of the cooling coefficient to within 2.5 per cent. For smaller values of the cooling coefficient the accuracy is still greater.

APPENDIX V

Calculation of Temperature Gradients in Friction Material

The friction material is a comparatively poor conductor of heat, and it is therefore legitimate to regard the pad or lining as infinitely thick for purposes of heat transmission calculation. We can then apply eq. (48) with $S_o = 0$, getting

$$\frac{d^2\bar{S}}{dx^2} = \frac{D_1 c_1 p}{k_1}\bar{S} = \mu^2 p\bar{S} \qquad (58)$$

where $\mu^2 = \dfrac{D_1 c_1}{k_1}$

c_1, k_1, D_1 being respectively the specific heat, conductivity and density of the friction material. The boundary condition is

$$\bar{S} = \int_0^\infty e^{-pt_2} S \, dt_2 \quad (x = 0) \qquad (59)$$

where S is the surface temperature of the disc, which is assumed to be known.

In practice, as we have seen, S is a complex function of time, and any solution using the actual law for S would be unwieldy, in fact probably quite unmanageale in most cases. To get a rough estimate we shall consider two simplified conditions.

The first case to be considered is that corresponding to a <u>gradual</u> application of brakes. A representative surface temperature - time curve for such a case is given in Fig. 2. We shall assume for the time being that this can be roughly approximated to by

$$S = S_{max}\left(1 - e^{qt_2}\right) \qquad (60)$$

which gives

$$\bar{S} = S_{max}\left(\frac{1}{p} - \frac{1}{p+q}\right) \quad (x = 0)$$

Solving eq. (58) and using the above boundary condition we get

$$\bar{S} = S_{max}\left(\frac{e^{-\mu x\sqrt{p}}}{p} - \frac{e^{-\mu x\sqrt{p}}}{p+q}\right) = \bar{S}_1 - \bar{S}_2$$

From tables of Laplace transforms, we find

$$S_1 = S_{max}\left(1 - \operatorname{erf}\frac{\mu x}{2\sqrt{t_2}}\right)$$

$$S_2 = \frac{S_{max}}{2} e^{-qt_2} \left[e^{-i\mu x\sqrt{q}} \left(1 - \text{erf} \left(\frac{\mu x}{2\sqrt{t_2}} - i\sqrt{qt_2} \right) \right.\right.$$

$$\left.\left. + e^{i\mu x\sqrt{q}} \left(1 - \text{erf} \left(\frac{\mu x}{2\sqrt{t_2}} + i\sqrt{qt_2} \right) \right) \right]$$

$$= \frac{S_{max}}{2} e^{-qt_2} \left[2 \cos \mu x\sqrt{q} - e^{-i\mu x\sqrt{q}} \text{erf} \left(\frac{\mu x}{2\sqrt{t_2}} - i.\sqrt{q}t_2 \right) \right.$$

$$\left. - e^{i\mu x\sqrt{q}} \text{erf} \left(\frac{\mu x}{2\sqrt{t_2}} + i\ qt_2 \right) \right]$$

with $S = S_1 - S_2$.

The temperature gradient immediately below the surface is given by

$$G = \left| \frac{dS}{dx} \right|_{x=0} = \left| \frac{dS_1}{dx} \right|_{x=0} - \left| \frac{dS_2}{dx} \right|_{x=0} = G_1 - G_2 \text{ say}$$

$$G_1 = \left| -\frac{S_{max}}{2} \frac{\mu}{\sqrt{t_2}} e^{-\frac{\mu^2 x^2}{\mu t2}} \right|_{x=0} = -\frac{S_{max}\,\mu}{2\sqrt{t_2}}$$

$$\frac{G_2}{S_{max}} = \tfrac{1}{2} e^{-qt2} \left(- 2\mu\sqrt{q}\sin\mu x\sqrt{q} + i\mu\sqrt{q}e^{-i\mu x\sqrt{q}} \text{ erf } (-i\ \sqrt{q}t_2) \right.$$

$$\left. - i\mu\sqrt{q}e^{i\mu x\sqrt{q}} \text{erf } (i\sqrt{qt_2}) - \frac{\mu}{\sqrt{t_2}} e^{qt2} \right) \text{ with } x = 0$$

i.e.,

$$\frac{G_2}{S_{max}} = \tfrac{1}{2}\ e^{-qt2} \left[i\mu\sqrt{q} \left(\text{erf}- i\sqrt{qt_2}\right) - \text{erf}(i\sqrt{qt_2}) - \frac{\mu . e^{qt_2}}{\sqrt{t_2}} \right]$$

Now $\text{erf } (iu) = - \text{ erf } - iu) = \frac{2i}{\sqrt{\pi}} \int_0^u e^{v^2}\, dv$

Hence $G_2 = S_{max} \left(\frac{2\mu\sqrt{q}}{\sqrt{\pi}} e^{-qt2} \int_0^{\sqrt{qt_2}} e^{u^2}\, du - \frac{\mu}{2\sqrt{t_2}} \right)$

and thus $G_1 - G_2 = \frac{2\mu\sqrt{q}}{\sqrt{\pi}} e^{-qt2} \int_0^{\sqrt{qt_2}} e^{u^2}\, du$

The gradient is a maximum when $\frac{dG}{dt_2} = 0$, i.e., when

$$e^{-qt_2} \int_0^{\sqrt{qt_2}} e^{u^2} \, du = \frac{1}{2\sqrt{qt_2}} \tag{61}$$

and the maximum gradient is given by

$$G_{max} = - \frac{S_{max}\,\mu}{\sqrt{\pi t_1}} \tag{62}$$

where t_1 is the solution of eq. (58).

Putting $qt_2 = y$, eq.(58) may be written

$$\Psi(y) = \frac{1}{\sqrt{y}} \int_0^{\sqrt{y}} e^{u^2} \, du - \frac{e^y}{2y} = 0 \tag{63}$$

Now $$e^{u^2} = \sum_{r=0} \frac{u^{2r}}{r!}$$

and therefore

$$\int_0^u e^{u^2} \, du = \sum_{r=0} \frac{u^{2r+1}}{r!\,(2r+1)} = u \sum_{r=0} \frac{u^{2r}}{r!\,(2r+1)}$$

Therefore

$$\frac{1}{\sqrt{y}} \int_0^{\sqrt{y}} e^{u^2} \, du = \sum_{r=0} \frac{y^r}{r!\,(2r+1)} = 1 + \sum_{r=1} \frac{y^r}{r!\,(2r+1)}$$

Again

$$\frac{e^y}{2y} = \frac{1}{2y} + \frac{1}{2} + \sum_{r=1} \frac{y^r}{2\,(r+1)!}$$

Hence

$$\Psi(y) = \frac{-1}{2y} + \frac{1}{2} + \sum_{r=1} \frac{y^r}{r!} \left(\frac{1}{2r+1} - \frac{1}{2(r+1)} \right)$$

$$= \frac{-1}{2y} + \frac{1}{2} + \frac{1}{2} \sum_{r=1} \frac{y^r}{r!\,(2r+1)\,(r+1)}$$

Eq.(60) then becomes

$$\frac{1}{y} = 1 + \sum_{r=1} \frac{y^r}{r!\,(r+1)(2r+1)}$$

$$= 1 + \frac{y}{2.3} + \frac{y^2}{3.5.2!} + \frac{y^3}{4.7.3!.} + \ldots.$$

Taking the first three terms only

$$\frac{1}{y} = 1 + \frac{y}{6} + \frac{y^2}{30}$$

The solution of this is $y = qt_2 = 0.856$

Inserting this value in equation (62), we get

$$G_{max} = \frac{-S_{max}\ \mu\sqrt{q}}{(0.856\pi)^{\frac{1}{2}}} = -\ 0.61\ S_{max}\ \mu\sqrt{q}$$

Now the initial rate of temperature rise is

$$q\ S_{max} = R_{max} \qquad \text{say.}$$

Hence

$$-\ G_{max} = 0.61\mu\ \sqrt{S_{max}\ R_{max}}$$

$$= 0.61\ \sqrt{\frac{D_1 c_1}{k_1}\ S_{max}\ R_{max}} \qquad (64)$$

where S_{max} is the peak surface temperature and R_{max} the initial rate of temperature rise, which for the assumed law (eq.(60)) is also the maximum rate of rise.

In practice, of course, the actual temperature - time relationship is substantially different from that given by eq.(60). We shall assume however, that eq.(64) still gives a sufficiently near approximation for temperature - time curves of the type of that of Fig.2., in which the initial rate of temperature growth is not infinite, provided that R_{max} is taken to be not the initial rate of increase (which may be zero) but the peak rate, i.e., the slope of the curve at the point of inflection.

Let us now consider the case of sudden application of brakes, and work out the conditions in the friction material at the beginning of the run. Under these conditions the temperature - time relationship is given by eq. (14), which for small values of t_2 approximates to

$$S = \frac{4H\ \sqrt{t_2}}{TA\sqrt{\pi cDk}} = Z\sqrt{t_2} \qquad \text{say.} \qquad (65)$$

Eq. (59) then becomes

$$\bar{S} = \int_0^{\infty} e^{-pt_2}\, 2\sqrt{t_2}\, dt_2 = \frac{\sqrt{\pi}}{2} \frac{Z}{p^{3/2}} \quad (x = 0)$$

Solving eq. (58) with the above boundary condition, we get

$$\bar{S} = \sqrt{\pi}\, \frac{Z\, e^{-\mu x\sqrt{p}}}{2p^{3/2}}$$

The solution of this can again be found from tables of Laplace transforms, and is

$$S = \sqrt{\pi} Z\, \sqrt{t_2} \left[\text{constant} - \int_0^{\frac{\mu x}{2\sqrt{t_2}}} (1 - \text{erf } u)\, du \right]$$

Since we are concerned with conditions near the surface, where x is small, we can write erf u = 0, and hence

$$S = \sqrt{\pi} Z\, \sqrt{t_2} \left(\text{constant} - \frac{\mu x}{2\sqrt{t_2}} \right)$$

The temperature gradient immediately below the surface is

$$\left| \frac{dS}{dx} \right|_{x = 0} = - \frac{Z\mu\sqrt{\pi}}{2} = - \sqrt{\frac{c_1 D_1}{k_1}}\, \frac{2H}{AT\sqrt{cDk}} \tag{66}$$

Note that this in independent of time, and that there is an instantaneous finite gradient as soon as the brakes are applied. Note also that since eq. (65) gives excessive temperatures when t_2 is finite but still small, during the early stages of the run the actual gradient decreases from the value given by eq. (66)

APPENDIX VI

Determination of Peak Temperatures and Gradients

We shall first consider the case of gradual application of brakes treated in Section 2.2. We have seen that in this case the surface temperature is given by equations (2) and (3) for $t < 1/3$ and $t > 1/3$ respectively.

First take eq. (2). Here S is a maximum when

$$2at^{\frac{1}{2}} - 4a^2t^{3/2} + \frac{8}{5}a^3t^{5/2} = 0$$

i.e., when

$$4a^2t^2 - 10at + 5 = 0$$

i.e., when

$$t = \frac{5}{4a}\left(1 \pm \frac{1}{\sqrt{5}}\right)$$

Now $at = \frac{t_1}{T} \times \frac{t_2}{t_1} = \frac{t_2}{T}$, and therefore $at < 1$, since $t_2 < T$.

Hence we must take the negative sign in the expression for t, i.e.,

$$t = \frac{5}{4a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

Now this expression can be valid only if $t < 1/3$, i.e., if $a > \frac{15}{4}\left(1 - \frac{1}{\sqrt{5}}\right)$
i.e., if $a > 2.07$.

Under these conditions, the maximum temperature occurs when

$$at = \frac{t_2}{T} = \frac{5}{4}\left(1 - \frac{1}{\sqrt{5}}\right) = 0.69,$$

i.e., at 69 per cent of the total time for the run.

Substituting the above value of at in eq. (2), we get

$$S_{max} = \frac{0.308 \times 8}{\sqrt{3}} \times \frac{hH}{TkA} \times \sqrt{\frac{0.69}{a}} = \frac{1.18hH}{TkA}\sqrt{\frac{kT}{cDh^2}} = \frac{1.18H}{A\sqrt{T}\ ckD}$$

The above equation gives the peak surface temperature if $a > 2.07$, but in practice is quite a close approximation if $a > 1$.

When $t > 1/3$, write $at = \frac{t_2}{T} = \theta$. Then eq. (3) can be written

$$S = \frac{4hH}{TkA} f(a, \theta)$$

and S is a maximum when $\theta = \theta_1$, where $f'(a, \theta_1) = 0$ (67)

Thus

$$S_{max} = \frac{4hH}{TkA} f(a, \theta_1)$$

It is convenient to write this

$$S_{max} = \frac{4hH}{TkAa}\left[a \times f(a, \theta_1)\right] = \frac{H}{AcDh}\left[4a \times f(a, \theta_1)\right]$$

$$= \frac{H}{AcDh} F_1(a) \qquad (68)$$

since θ_1 is a function of a.

The above expression is valid for all values of a if we put $F_1(a) = 1.18\sqrt{a}$ when $a > 2.07$.

For values of a less than 2.07, we must solve eq. (67). Now $f'(a, \theta)$ is a polynomial in θ which is easily derived from eq. (3) and need not be given here. Eq. (67) is solved by numerical methods, giving the value of θ appropriate to any chosen value of a. These values are inserted in eq. (15), giving $F_1(a)$, which is plotted in Fig. 4.

The rate of temperature rise is given by $R = \frac{dS}{dt_2}$, and this rate is a maximum when

$$\frac{dR}{dt_2} = \frac{d^2S}{dt_2^2} = 0$$

We can write eq. (2)

$$S = \frac{8H}{\sqrt{3T}\,A\sqrt{ckD}}\sqrt{\theta}\left(\frac{4}{3}\theta - \frac{8}{5}\theta^2 + \frac{16}{35}\theta^3\right)$$

and therefore

$$R = \frac{dS}{dt_2} = \frac{dS}{d\theta}\frac{d\theta}{dt_2} = \frac{1}{T}\frac{dS}{d\theta}$$

$$= \frac{8H}{\sqrt{3}\,T^{3/2}\,A\sqrt{ckD}}\left(2\theta^{\frac{1}{2}} - 4\theta^{3/2} + \frac{8}{5}\theta^{5/2}\right) \qquad (69)$$

R is a maximum when $\frac{dR}{d\theta} = 0$, i.e. when

$$\theta^{-\frac{1}{2}} - 6\theta^{\frac{1}{2}} + 4\theta^{3/2} = 0$$

This gives $\theta = \frac{3 \pm \sqrt{5}}{4}$. Here again the negative sign must be taken, giving $\theta = \frac{3 - \sqrt{5}}{4} = 0.191$.
Again this is valid provided $t = \frac{\theta}{a} < \frac{1}{3}$, i.e., provided $a > 3 \times 0.191 = 0.573$.

Putting $\theta = 0.191$ in eq. (6?), we get

$$R_{max} = \frac{H}{T^{3/2} A\sqrt{ckD}} \times \frac{8}{\sqrt{3}} \times 0.565 = \frac{2.61\ H}{A\ T^{3/2}\sqrt{ckD}}$$

Now we have shown above that for $a > 2.07$

$$S_{max} = \frac{1.18H}{a\sqrt{T}\sqrt{ckD}}$$

Hence for $a > 2.07$ the product $\sqrt{R_{max}\ S_{max}}$ is

$$\frac{H}{TA\sqrt{ckD}} \times \sqrt{2.61 \times 1.18} = \frac{1.76H}{TA\sqrt{ckD}}$$

Inserting this in eq. (61) gives eq. (30).

The above expression is in fact quite a close approximation down to $a = 1$.

For $a < 0.573$ we must use eq. (3), and again we may write this

$$S = \frac{H}{AcDh} \left[4a\ f(a,\ \theta)\right]$$

and therefore

$$R = \frac{1}{T}\frac{dS}{d\theta} = \frac{H}{ATcDh} \left[4a\ f'(a,\ \theta)\right] \tag{70}$$

R is a maximum when $f''(a,\ \theta) = 0$ (71)

$f''(a,\ \theta)$ is a polynomial in θ which is easily derived from eq. (3), and eq. (71) can therefore be solved for any chosen value of a, giving the corresponding value of θ. Inserting this value in eq. (70) we may write

$$R_{max} = \frac{H}{ATcDh} G(a) \tag{72}$$

where $G(a)$ is a function of a which can be calculated by the method just explained, using numerical procedures. Eq. (27) is valid for all values of a if we put $G(a) = 2.61\sqrt{a}$ when $a > 0.573$.

The product $\sqrt{R_{max}\ S_{max}}$ is then given by

$$\frac{H}{\sqrt{T}\ AcDh}\sqrt{F_1(a)\ G(a)} = \frac{H}{\sqrt{T}\ AcDh}\ \Phi\ (a)\ \text{say}$$

Inserting this in eq.(64) gives eq.(29). $\Phi(a)$ is plotted against a in Fig.4. The above expression is valid for all values of a if we put $\Phi(a) = 1.76\sqrt{a}$ when $a > 2.07$.

Let us now calculate peak temperatures for the constant torque case. Here the temperature is given by eq.(7), for which the approximations (10), (11), (12) and (13) will be used.

The procedure is essentially similar to that followed above. It is easily shown that if $a < 2/3$ the peak temperature occurs in the range in which eq's (12) and (13) are valid, and is given by

$$S_{max} = \frac{H}{AcDh}\left(1 + \frac{a^2}{2}\right)$$

When $a > \frac{2}{3}$, we must use eq's (10) and (11). We have then

$$S = \frac{4}{\sqrt{\pi}}\frac{hH}{tkA}\left[t^{\frac{1}{2}} + \frac{1}{3}t^{3/2} - \frac{2}{3}a\left(t^{3/2} + \frac{1}{5}t^{5/2}\right)\right] \qquad (73)$$

The peak temperature occurs when $\frac{dS}{dt} = 0$, i.e., when

$$3 + 3t - a(6t + 2t^2) = 0$$

i.e., when

$$t = \frac{1}{4a}\left(\sqrt{36a^2 - 12a + 9} - 6a + 3\right) = t_o \quad \text{say}$$

Inserting this value of t in (68) gives

$$S_{max} = \frac{4}{\sqrt{\pi}}\ \frac{hH}{TkA}\ f_2(a,t_o) = \frac{4}{\sqrt{\pi}}\frac{hH}{TkAa}\left[a\ f_2(a,t_o)\right]$$

$$= \frac{H}{AcDh}\ F_2(a) \qquad (74)$$

$F_2(a)$ is plotted against a in Fig.4.

For large values of a we have $t_o \approx \frac{1}{2a}$

Hence $$S_{max}\ \frac{4}{\sqrt{\pi}}\frac{hH}{TkA}\frac{1}{\sqrt{2a}}\left[1 + \frac{1}{6a} - \frac{2a}{3} \times \frac{1}{2a}\left(1 + \frac{1}{10a}\right)\right]$$

$$= \frac{4}{\sqrt{\pi}}\frac{hH}{TkA} \times \frac{1}{\sqrt{2a}}\left(\frac{2}{3} + \frac{2}{15a}\right)$$

$$\approx \frac{8}{3\sqrt{2\pi}}\ \frac{hH}{TkA\sqrt{a}} = \frac{1.065H}{A\sqrt{T}\sqrt{ckD}} \qquad (75)$$

Eq.(16) is valid for all values of a if $F_2(a)$ is taken at $1 + a^{2/2}$ for $a < 2/3$, and $F_2(a) \rightarrow 1.065^2\sqrt{a}$ as $a \rightarrow \infty$. The convergence to the "thick disc" expression (75) is however very much slower than in the case of gradual application of brakes, there still being an appreciable difference when a is as high as 2.0.

We shall now determine peak surface temperatures in the constant torque case for a cooled brake. We shall deal only with the case of a comparatively thin disc, which lends itself to simple analysis (in any event most practical brakes come within this category). If the disc is comparatively thin, the peak temperature occurs when t is fairly large, and we can then use eq.(23), viz.

$$S = \frac{2hH}{TkAn}\left(1 + \frac{a}{n} - at - \frac{2}{\alpha - \beta}\, e^{\beta t}\,(a - \beta)\right)$$

The maximum temperature occurs when $\frac{dS}{dt} = 0$, i.e., when

$$- a - \frac{2(a - \beta)}{\alpha - \beta}\,\beta e^{\beta t} = 0$$

i.e., when

$$t = \frac{1}{\beta}\log\frac{-a(\alpha - \beta)}{2\beta(a - \beta)}$$

This gives

$$S_{max} = \frac{2hH}{TkAn}\left[1 + a\left(\frac{1}{n} + \frac{1}{} - \frac{1}{\beta}\log\frac{-a(\alpha - \beta)}{2\beta(a - \beta)}\right]\right.$$

This can be written

$$S_{max} = \frac{2hH}{TkAn}\left\{1 + \frac{a}{n}\left(1 + \frac{n}{\beta} - \frac{n}{\beta}\log\frac{(\alpha - \beta)}{-2\beta}\right)\right.$$

$$\left. - \frac{a}{n}\left[-\frac{n}{\beta}\log\left(1 - \frac{n}{a} \times \frac{\beta}{n}\right)\right]\right\}$$

Put

$$\frac{a}{n} = \frac{cDh^2}{kT} \times \frac{k}{bh} = \frac{cDh}{bT} = \frac{1}{\eta}$$

Then

$$\frac{2hH}{tKAn} = \frac{2hHk}{TkAbh} = \frac{2H}{bTA} = \frac{2H}{AcDh}\,\frac{cDh}{bT} = \frac{2}{\eta}\,\frac{H}{AcDh}$$

We thus obtain eq.(24).

APPENDIX VII

Evaluation of Transforms

We shall use the notation f(p) = £ g(t) to denote that f(p) is the Laplace transform of g(t).

A standard result is

$$\frac{1}{p(1+p)^{\frac{1}{2}}} = \pounds(\operatorname{erf}\sqrt{t})$$

Therefore

$$\frac{1}{p(1+p)^{\frac{1}{2}}} = \pounds\left[\int_0^t \operatorname{erf}\sqrt{t}\,dt\right]$$

$$= \pounds\left[t\operatorname{erf}\sqrt{t} - \int_0^t t\frac{d}{dt}(\operatorname{erf}\sqrt{t})\,dt\right]$$

$$= \pounds\left[t\operatorname{erf}\sqrt{t} - \frac{1}{\sqrt{\pi}}\int_0^t t(t^{-\frac{1}{2}}e^{-t})dt\right]$$

$$= \pounds\left[t\operatorname{erf}\sqrt{t} - \frac{1}{\sqrt{\pi}}\int_0^t t^{\frac{1}{2}}e^{-t}dt\right]$$

$$= \pounds\left[t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2\sqrt{\pi}}\int_0^t t^{-\frac{1}{2}}e^{-t}dt\right]$$

$$= \pounds\left[t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right]$$

Using the above result

$$I = \frac{1}{p3(1+p)^{\frac{1}{2}}} = \pounds\left[\int_0^t \left(t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right)dt\right]$$

$$= \pounds\left[t\left(t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right)\right.$$

$$\left. - \int_0^t t\operatorname{erf}\sqrt{t}\,dt\right]$$

$$= \pounds\left[t\left(t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right)\right.$$

$$- \int_0^t \left(t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right)dt$$

$$\left. - \frac{1}{2}\left(t\operatorname{erf}\sqrt{t} + \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}} - \frac{1}{2}\operatorname{erf}\sqrt{t}\right) + \int_0^t \frac{\sqrt{t}\,e^{-t}}{\sqrt{\pi}}\,dt\right]$$

Hence

$$2I = \mathcal{L}\left[t^2 \operatorname{erf}\sqrt{t} + \frac{t^{3/2}e^{-t}}{\sqrt{\pi}} - \tfrac{1}{2}t \operatorname{erf}\sqrt{t} - \frac{\frac{1}{2}\sqrt{t}\, e^{-t}}{\sqrt{\pi}}\right.$$

$$\left. + \tfrac{1}{4}\operatorname{erf}\sqrt{t} - \frac{\sqrt{t}\, e^{-t}}{\sqrt{\pi}} + \tfrac{1}{2}\operatorname{erf}\sqrt{t}\right]$$

Thus

$$I = \mathcal{L}\left[\tfrac{1}{2}\operatorname{erf}\sqrt{t}\,(t^2 - t + \tfrac{3}{4}) + \frac{t^{3/2}e^{-t}}{2\sqrt{\pi}} \quad \frac{3\sqrt{t}\, e^{-t}}{4\sqrt{\pi}}\right]$$

Again standard results are

$$\frac{1}{p - a} = \mathcal{L}(e^{at})$$

and

$$\frac{1}{(1 + p)^{\frac{1}{2}}} = \mathcal{L}\left[\frac{e^{-t}}{\sqrt{\pi t}}\right]$$

Hence by Borel's multiplication theorem

$$(p - a)^{-1}\,(1 + p)^{-\frac{1}{2}} = \mathcal{L}\left[\int_0^t \frac{e^{-T}}{\sqrt{\pi T}}\, e^{a}\,(t - T)\, dT\right]$$

$$= \mathcal{L}\left[\frac{e^{at}}{\sqrt{\pi}}\int_0^t \frac{e^{-(1 + a)T}dT}{\sqrt{T}}\right]$$

$$= \mathcal{L}\left[\frac{e^{at}\operatorname{erf}\,(1 + a)t}{(1 + a)^{\frac{1}{2}}}\right]$$

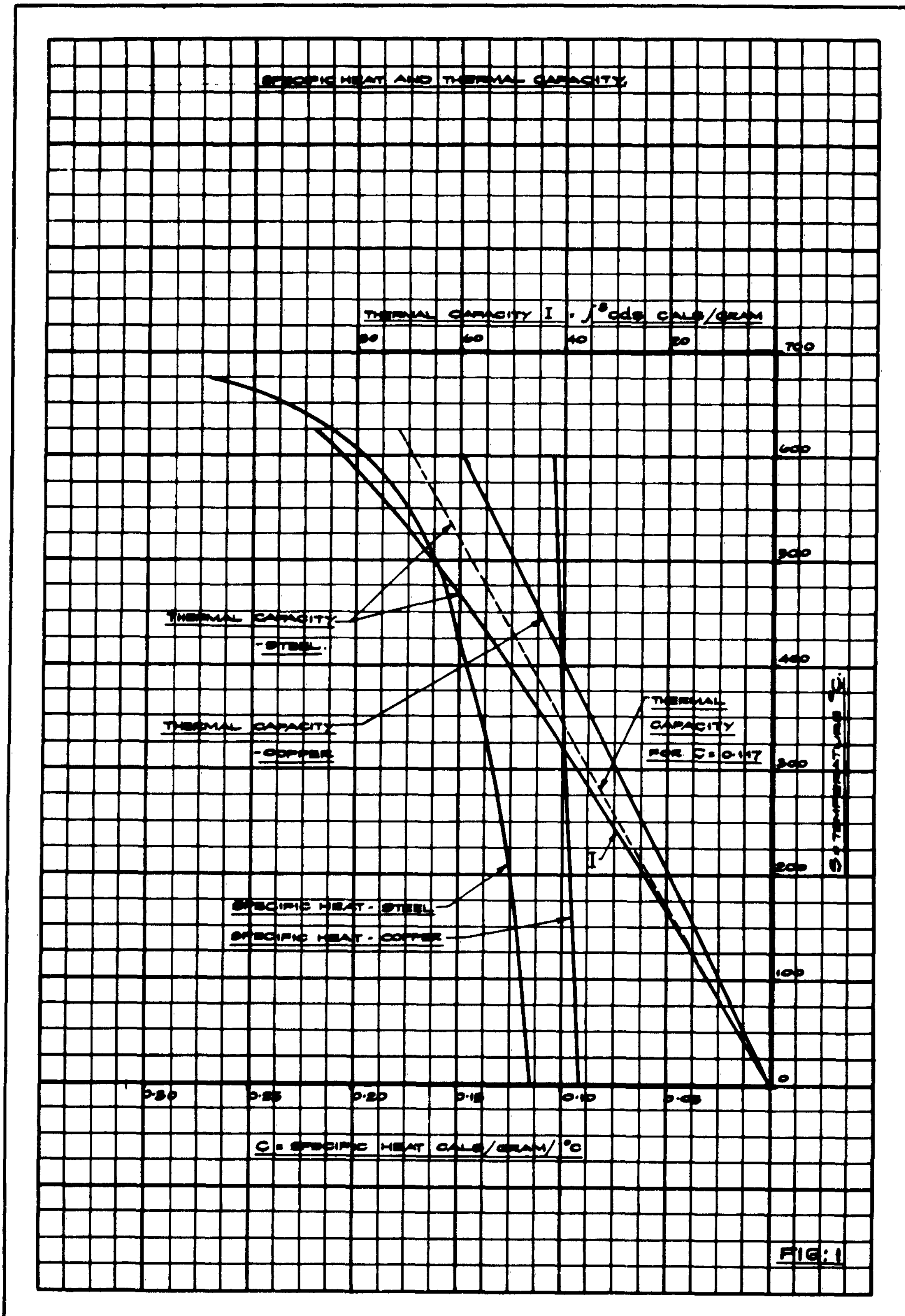

SPECIFIC HEAT AND THERMAL CAPACITY
THERMAL CAPACITY I = ∫θ c dθ CALS/GRAM
80
60
40
20
700
600
500
400
300
200
100
0
θ = TEMPERATURE °C
THERMAL CAPACITY - STEEL.
THERMAL CAPACITY - COPPER
THERMAL CAPACITY FOR C = 0·117
I
SPECIFIC HEAT - STEEL
SPECIFIC HEAT - COPPER
0·30
0·25
0·20
0·15
0·10
0·05
C = SPECIFIC HEAT CALS/GRAM/°C

FIG: 1

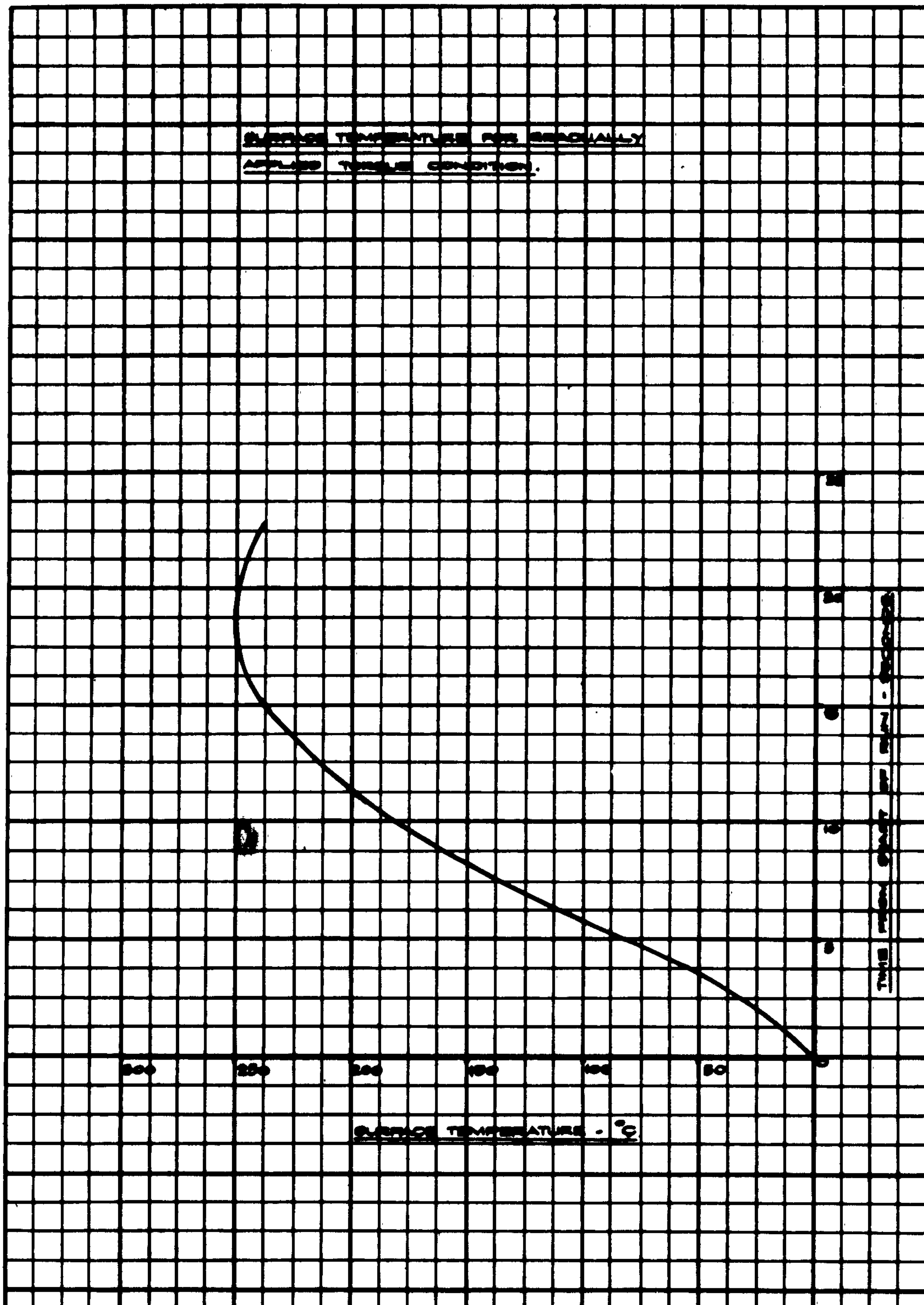
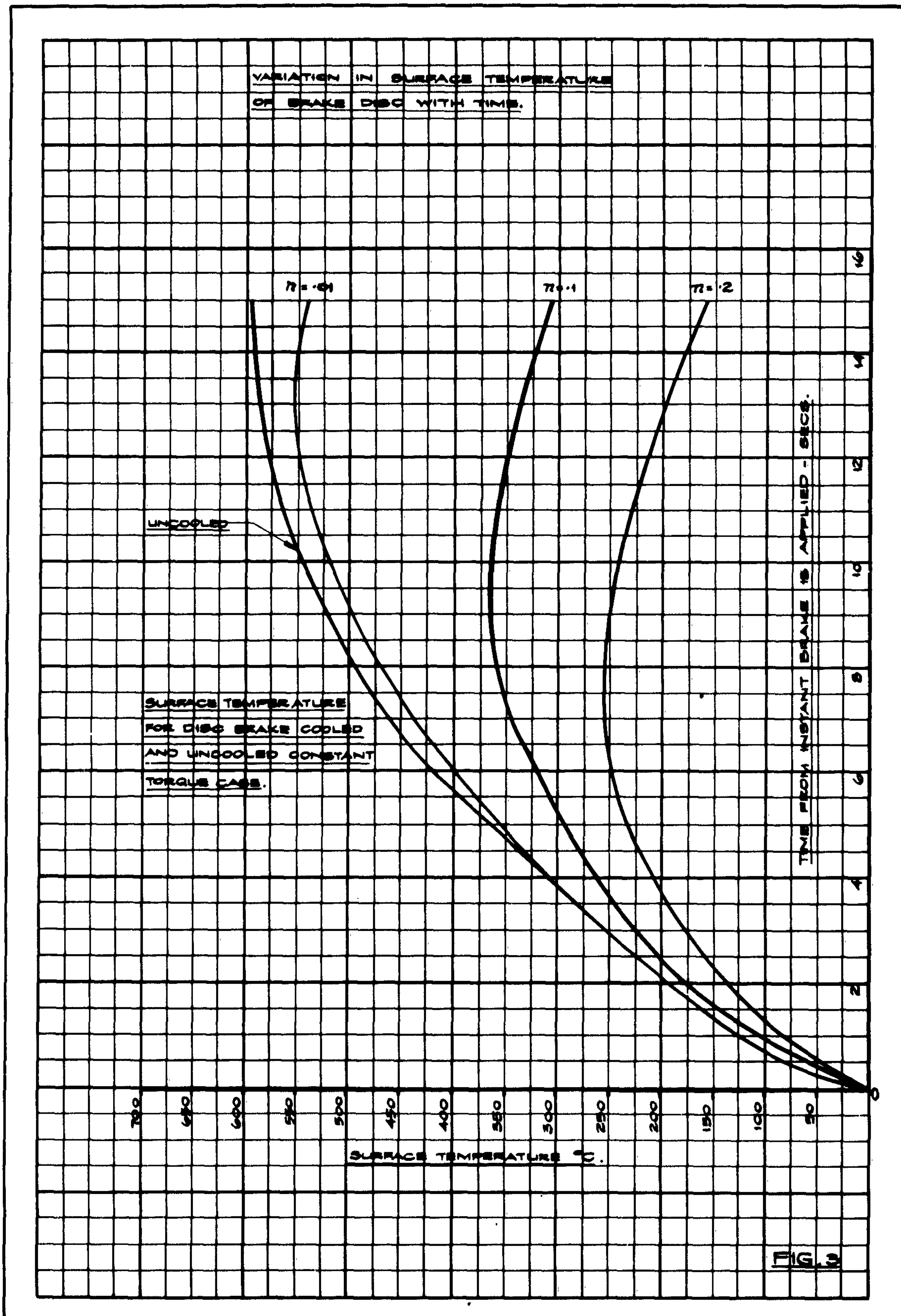

SURFACE TEMPERATURE FOR GRADUALLY
APPLIED TORQUE CONDITION.
300
250
200
150
100
50
0
SURFACE TEMPERATURE - °C
TIME FROM START OF RUN - SECONDS

FIG. 2.



VARIATION IN SURFACE TEMPERATURE OF BRAKE DISC WITH TIME.
η = ·01
η = ·1
η = ·2
UNCOOLED
SURFACE TEMPERATURE FOR DISC BRAKE COOLED AND UNCOOLED CONSTANT TORQUE CASE.
TIME FROM INSTANT BRAKE IS APPLIED - SECS.
0
2
4
6
8
10
12
14
16
700
650
600
550
500
450
400
350
300
250
200
150
100
50
SURFACE TEMPERATURE °C.

FIG. 3

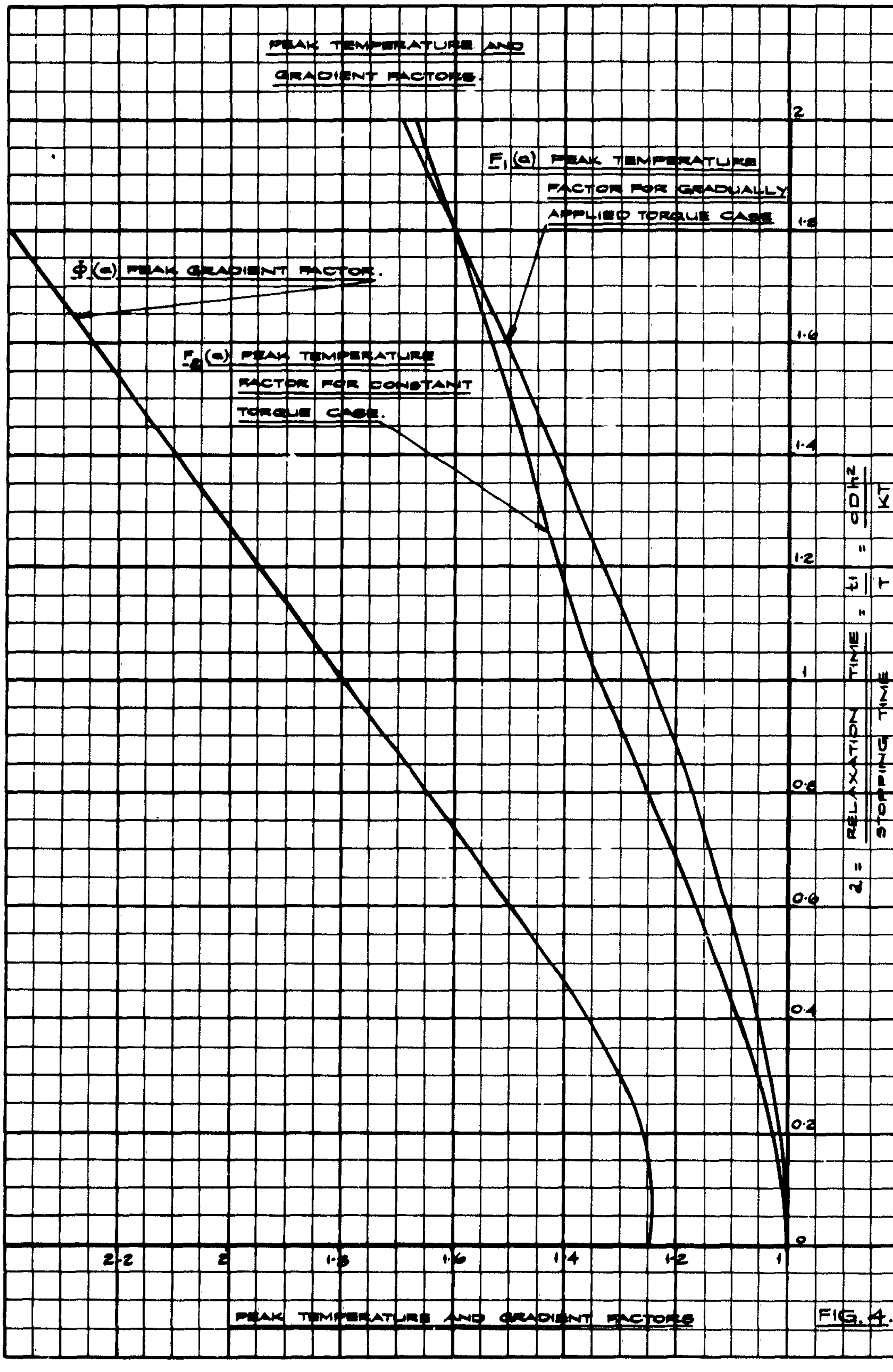

PEAK TEMPERATURE AND
GRADIENT FACTORS.
$F_1(a)$ PEAK TEMPERATURE
FACTOR FOR GRADUALLY
APPLIED TORQUE CASE
$\Phi(a)$ PEAK GRADIENT FACTOR.
$F_2(a)$ PEAK TEMPERATURE
FACTOR FOR CONSTANT
TORQUE CASE.
2
1·8
1·6
1·4
1·2
1
0·8
0·6
0·4
0·2
0
2·2
2
1·8
1·6
1·4
1·2
1
$a = \frac{\text{RELAXATION TIME}}{\text{STOPPING TIME}} = \frac{t_1}{T} = \frac{cDh^2}{KT}$
PEAK TEMPERATURE AND GRADIENT FACTORS

FIG. 4.

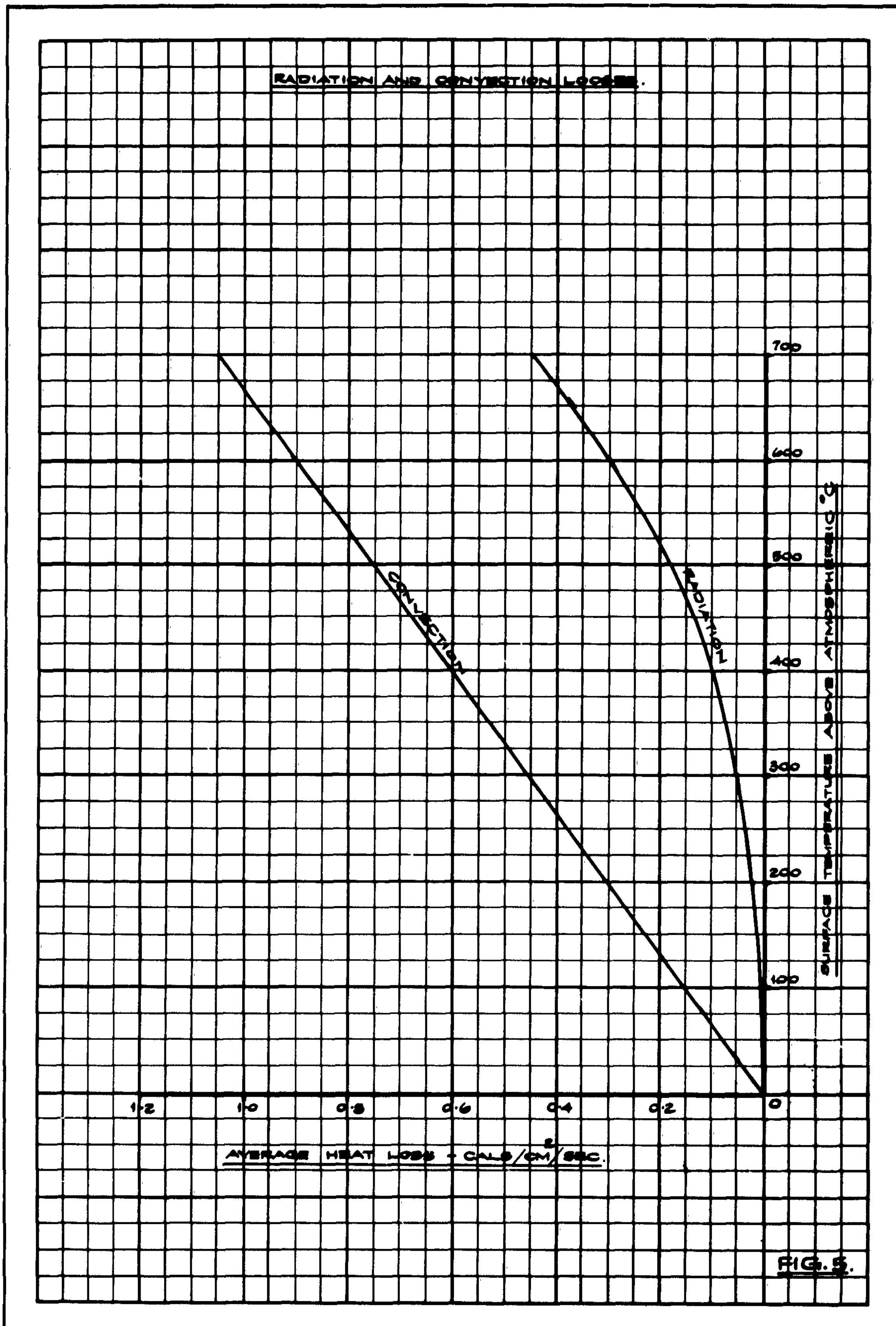
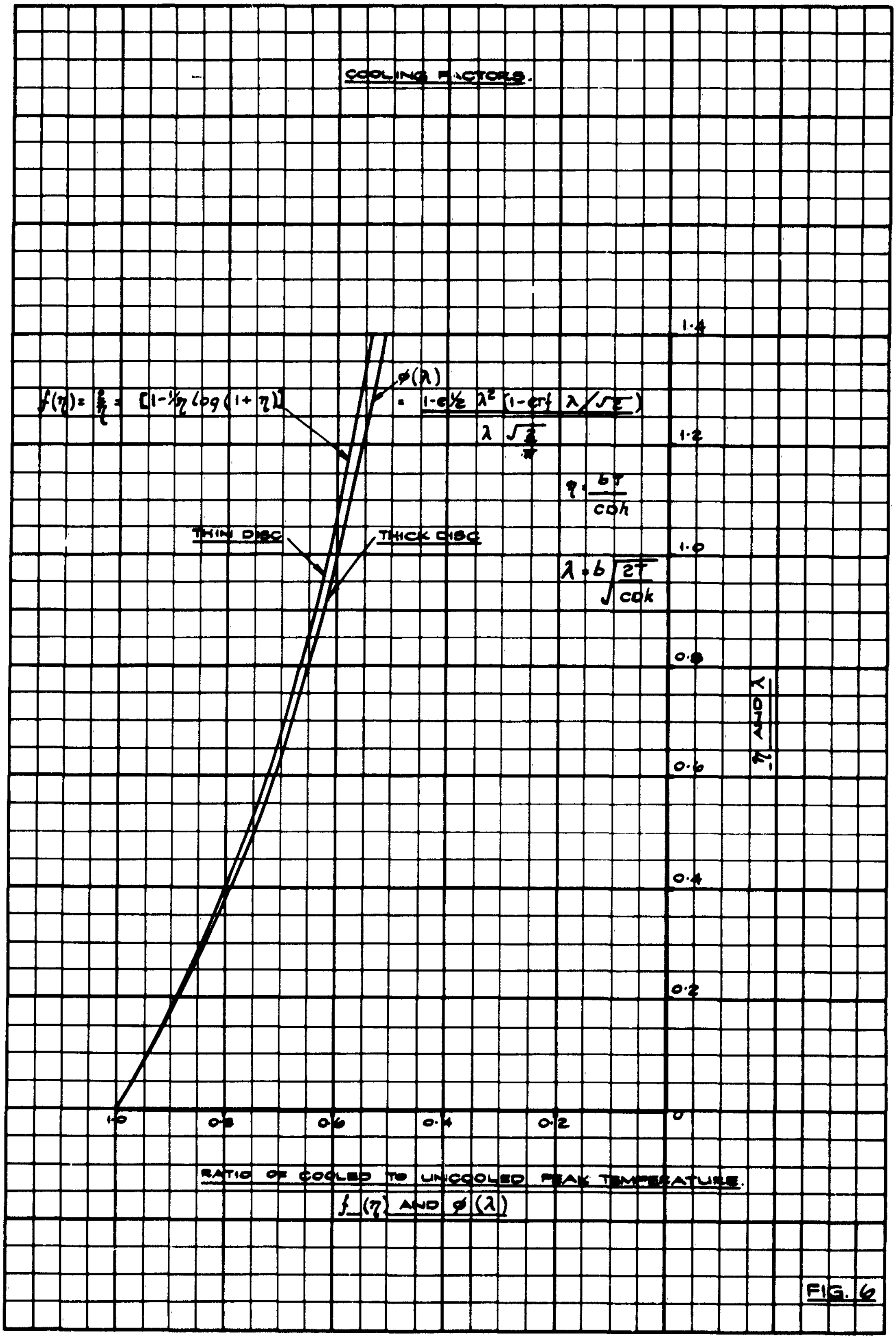

RADIATION AND CONVECTION LOSSES.
CONVECTION
RADIATION
700
600
500
400
300
200
100
0
SURFACE TEMPERATURE ABOVE ATMOSPHERIC °C
1·2
1·0
0·8
0·6
0·4
0·2
AVERAGE HEAT LOSS - CALS/CM²/SEC.

FIG. 5.


COOLING FACTORS.
$f(\eta) = \frac{\xi}{\eta} = [1 - 1/\eta \log(1+\eta)]$
$\phi(\lambda)$
$= \frac{1 - e^{1/2 \lambda^2}(1 - \mathrm{erf}\ \lambda/\sqrt{2})}{\lambda\sqrt{\frac{2}{\pi}}}$
$\eta = \frac{bT}{c\rho h}$
$\lambda = b\sqrt{\frac{2T}{c\rho k}}$
THIN DISC
THICK DISC
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
η AND λ
1·0
0·8
0·6
0·4
0·2
RATIO OF COOLED TO UNCOOLED PEAK TEMPERATURE.
$f(\eta)$ AND $\phi(\lambda)$
FIG. 6

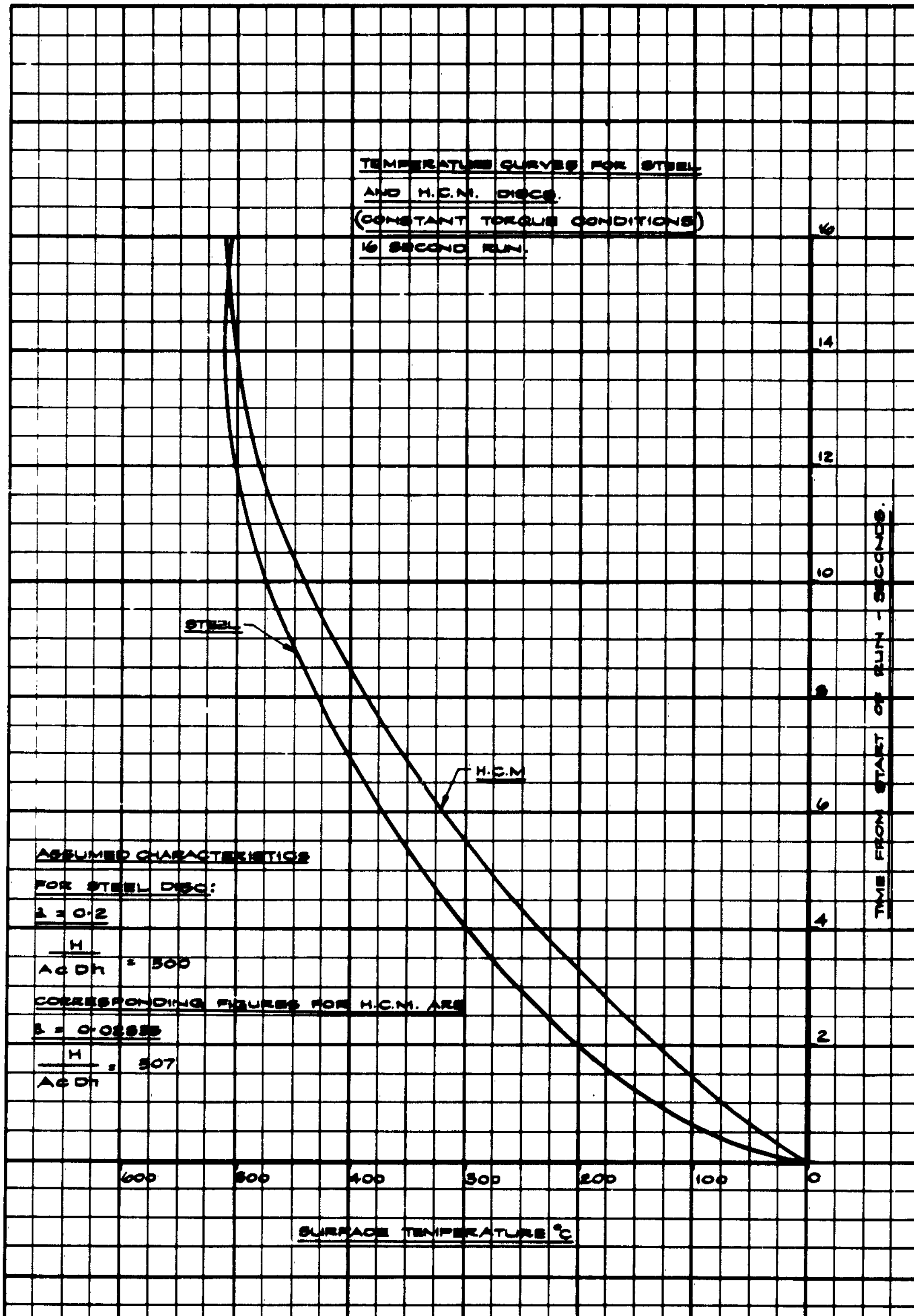

TEMPERATURE CURVES FOR STEEL
AND H.C.M. DISCS.
(CONSTANT TORQUE CONDITIONS)
16 SECOND RUN.
STEEL
H.C.M
ASSUMED CHARACTERISTICS
FOR STEEL DISC:
= 0·2
H / Ac Dh = 500
CORRESPONDING FIGURES FOR H.C.M. ARE
= 0·02555
H / Ac Dh = 507
600
500
400
300
200
100
0
2
4
6
8
10
12
14
16
SURFACE TEMPERATURE °C
TIME FROM START OF RUN - SECONDS.

FIG. 7.

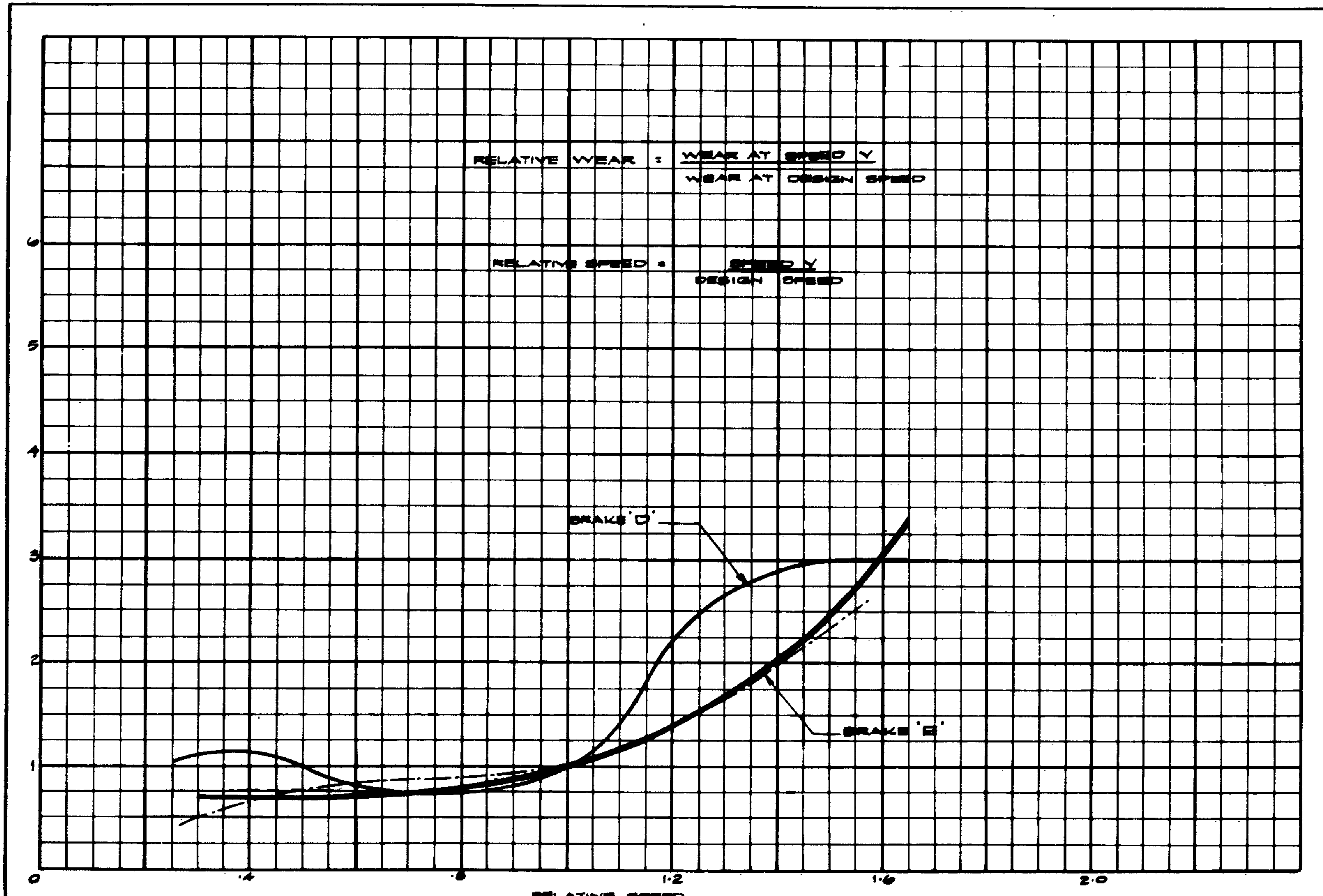

RELATIVE WEAR = WEAR AT SPEED V / WEAR AT DESIGN SPEED
RELATIVE SPEED = SPEED V / DESIGN SPEED
BRAKE 'D'
BRAKE 'E'
RELATIVE WEAR
RELATIVE SPEED
0
·4
·8
1·2
1·6
2·0
1
2
3
4
5
6

FIG. 8.

TEST CONDITIONS AND RESULTS.											REMARKS
TEST Nº	KE ft lb.	R.P.M.	I lb ft²	PAD SPEED ft/min	AVERAGE OP. PRESS P.S.I.	AVERAGE STOP TIME SEC.	AVERAGE TORQUE LB. IN.	AVERAGE H.P.	AVERAGE μ	AVERAGE WEAR PER PAD	EACH TEST COMPRISED 25 STOPS.
1	272000	480	6980	645	160	31·60	4,180	13·60	·241	·0023	TEST MADE FROM 100% KE AT APPROX. 35% NORMAL R.P.M AND USING NORMAL TORQUE - H.C.M. DISC. 3·25 lb.
2	272000	480	6980	645	480	10·90	12,350	47·0	·258	·0045	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
3	272000	975	1670	1310	160	15·70	4075	31·4	·230	·0015	TEST MADE FROM 100% KE AT APPROX 68% NORMAL R.P.M AND USING NORMAL TORQUE - H.C.M. DISC. 3·25 lb.
4	272000	975	1670	1310	240	10·60	6025	46·5	·236	·0025	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
5	272000	1440	769	1930	160	10·50	4100	47·0	·241	·0020	TEST MADE FROM 100% KE AND USING NORMAL TORQUE AND NORMAL R.P.M. - H.C.M DISC. 3·25 lb.
6	272000	1730	495	2400	160	8·60	4080	57·5	·238	·0050	TEST MADE FROM 100% KE AT APPROX 124% NORMAL R.P.M. AND USING NORMAL TORQUE - H.C.M. DISC 3·25 lb
7	272000	1730	495	2400	130	10·60	3280	46·5	·267	·0080	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
8	272000	2380	281	3190	160	6·30	4150	78·0	·244	·0060	TEST MADE FROM 100% KE AT APPROX 165% NORMAL R.P.M. AND USING NORMAL TORQUE - H.C.M. DISC 3·25 lb.
9	272000	2380	281	3190	100	10·60	2470	46·5	·232	·0045	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
10	197000	1440	557	1930	160	7·60	4100	47·0	·241	·0015	TEST MADE FROM 73% KE AT NORMAL R.P.M. AND USING NORMAL TORQUE - H.C.M. DISC - 3·25 lb
11	197000	1440	557	1930	120	10·60	2860	33·7	·232	·0015	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
12	343000	1440	967	1930	160	13·40	4075	46·5	·230	·0165	TEST MADE FROM 126% KE AT NORMAL R.P.M. AND USING NORMAL TORQUE - H.C.M. DISC. - 3·25 lb.
13	343000	1440	967	1930	210	10·50	5200	59·0	·238	·0130	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
14	408000	1410	1196	1890	170	16·80	3960	44·5	·219	·0450	TEST MADE FROM 150% KE AT APPROX NORMAL R.P.M AND NORMAL TORQUE. - H.C.M. DISC 3·25 lb.
15	408000	1410	1196	1890	280	10·70	6800	69·5	·208	·0540	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
16	272000	1440	769	1930	200	10·70	4085	46·0	·199	·0230	TEST MADE FROM 100% KE USING NORMAL TORQUE AND NORMAL R.P.M. - CHROMED STEEL DISC - 3·64 lb.
17	272000	1440	769	1930	150	10·40	4140	47·0	·259	·0310	TEST MADE FROM 100% KE USING NORMAL TORQUE AND NORMAL R.P.M. - UNCHROMED STEEL DISC - 2·90 lb.

TESTS ON BRAKE "D".

FIG. 9.

TEST CONDITIONS AND RESULTS.											REMARKS.
TEST N°	KE ft lb	R.P.M.	I lb ft^2	PAD SPEED ft/min	AVERAGE OF PRESS P.S.I.	AVERAGE STOP TIME SEC.	AVERAGE TORQUE LB. IN	AVERAGE H.P.	AVERAGE μ	AVERAGE WEAR PER PAD	EACH TEST COMPRISED 25 STOPS.
1	1415000	435	45200	1000	700	49·50	15200	52	·298	·008	TEST MADE FROM 100% KE AT 32% NORMAL R.P.M. USING NORMAL TORQUE. – H.C.M. DISC – 17 lb.
2	1415000	435	45200	1000	1500	25·50	28,800	100	·268	·016	AS ABOVE BUT USING MAXIMUM BRAKE PRESSURE TO DECREASE STOPPING TIME
3	1415000	895	10380	2060	700	24·60	14,750	105	·285	·012	TEST MADE FROM 100% KE AT 66% NORMAL R.P.M. USING NORMAL TORQUE – H.C.M. DISC. 17 lb.
4	1415000	895	10380	2060	1100	16·50	22,000	156	·274	·015	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
5	1415000	1360	4490	3130	700	16·00	14,900	161	·291	·014	TEST MADE FROM 100% KE USING NORMAL R.P.M. AND NOMAL TORQUE – H.C.M. DISC – 17 lb.
6	1415000	1840	2460	4240	780	12·30	14,350	210	·262	·026	TEST MADE FROM 100% KE USING 135% NORMAL R.P.M. AND NORMAL TORQUE – H.C.M. DISC 17 lb.
7	1415000	1840	2460	4240	600	16·40	10,750	157	·246	·019	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
8	1415000	2480	1350	5700	800	9·20	14,200	280	·244	·058	TEST MADE FROM 100% KE AT 182% NORMAL R.P.M. USING NORMAL TORQUE – H.C.M. DISC. – 17 lb.
9	1415000	2480	1350	5700	500	15·50	8,460	166	·232	·042	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
10	1060000	1390	3220	3200	800	11·50	15,200	167	·260	·016	TEST MADE FROM 75% KE AT APPROX NORMAL R.P.M. USING NORMAL TORQUE. H.C.M. DISC. – 17 lb.
11	1060000	1390	3220	3200	600	15·80	11,075	122	·253	·012	AS ABOVE BUT USING DECREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
12	1720000	1310	5910	3010	850	20·00	15,000	157	·242	·070	TEST MADE FROM 122% K.E. AT APPROX NORMAL R.P.M. AND USING NORMAL TORQUE – H.C.M. DISC. 17 lb
13	1720000	1310	5910	3010	1100	15·50	19,400	202	·242	·078	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
14	2130000	1340	6980	3080	900	23·70	15,400	164	·254	·154	TEST MADE FROM 150% KE AT APPROX NORMAL R.P.M. AND USING NORMAL TORQUE H.C.M. DISC – 17 lb
15	2130000	1340	6980	3080	1400	15·80	23,000	246	·225	·187	AS ABOVE BUT USING INCREASED TORQUE TO PRODUCE NORMAL STOPPING TIME.
16	1415000	1360	4490	3130	850	16·70	14,300	154	·231	·024	TEST MADE FROM 100% KE USING NORMAL R.P.M. AND NORMAL TORQUE – CHROMED BRONZE DISC – 17·50 lb
17	1415000	1360	4490	3130	1000	16·40	14,600	157	·200	·048	TEST MADE FROM 100% KE USING NORMAL R.P.M. AND NORMAL TORQUE – UNCROMED BRONZE DISC – 17·0 lb

TESTS ON BRAKE "E"

FIG. 10.

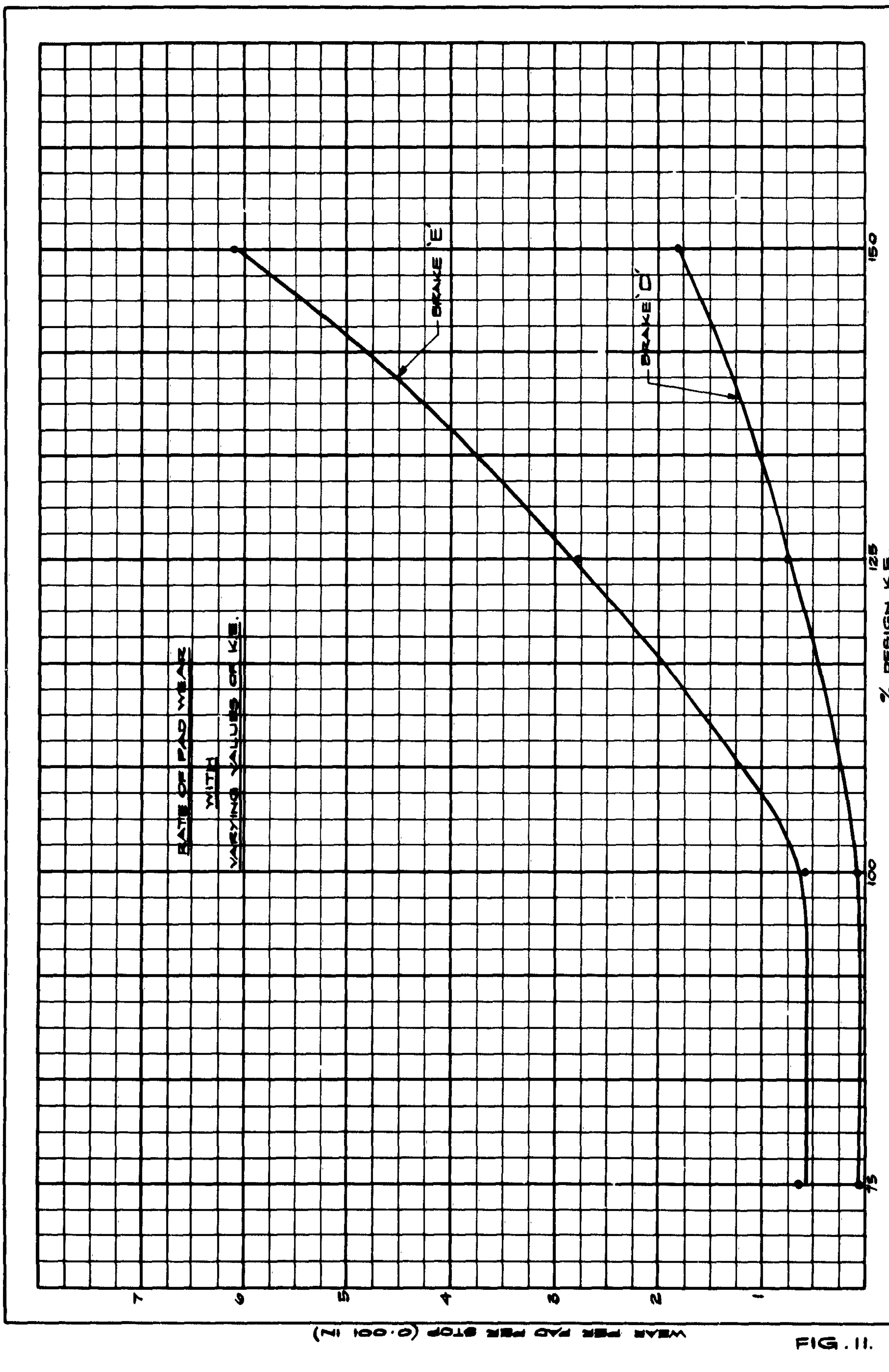
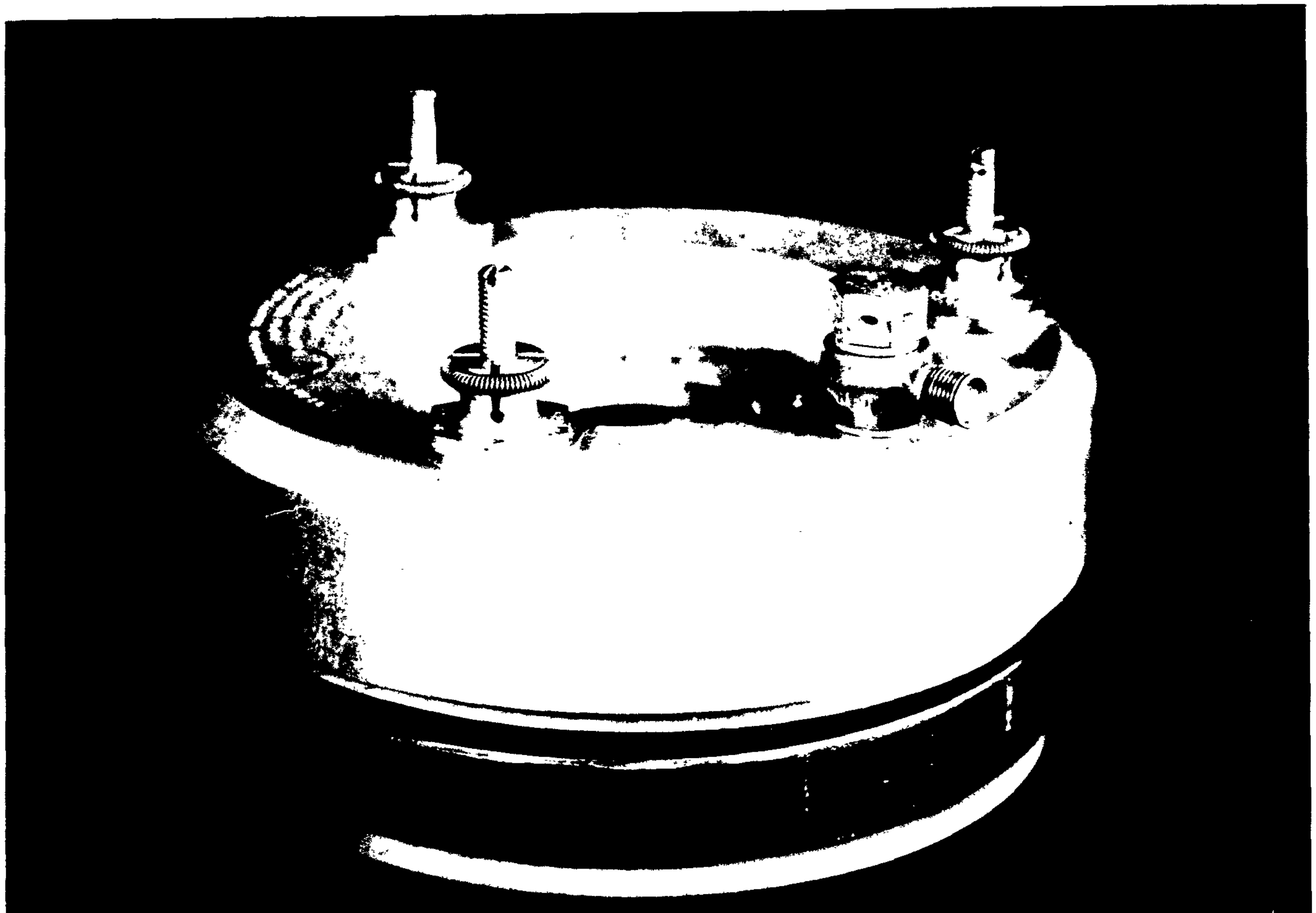

RATE OF PAD WEAR
WITH
VARYING VALUES OF K.E.
BRAKE 'E'
BRAKE 'D'
75
100
125
150
% DESIGN K.E.
1
2
3
4
5
6
7
WEAR PER PAD PER STOP (0.001 IN)

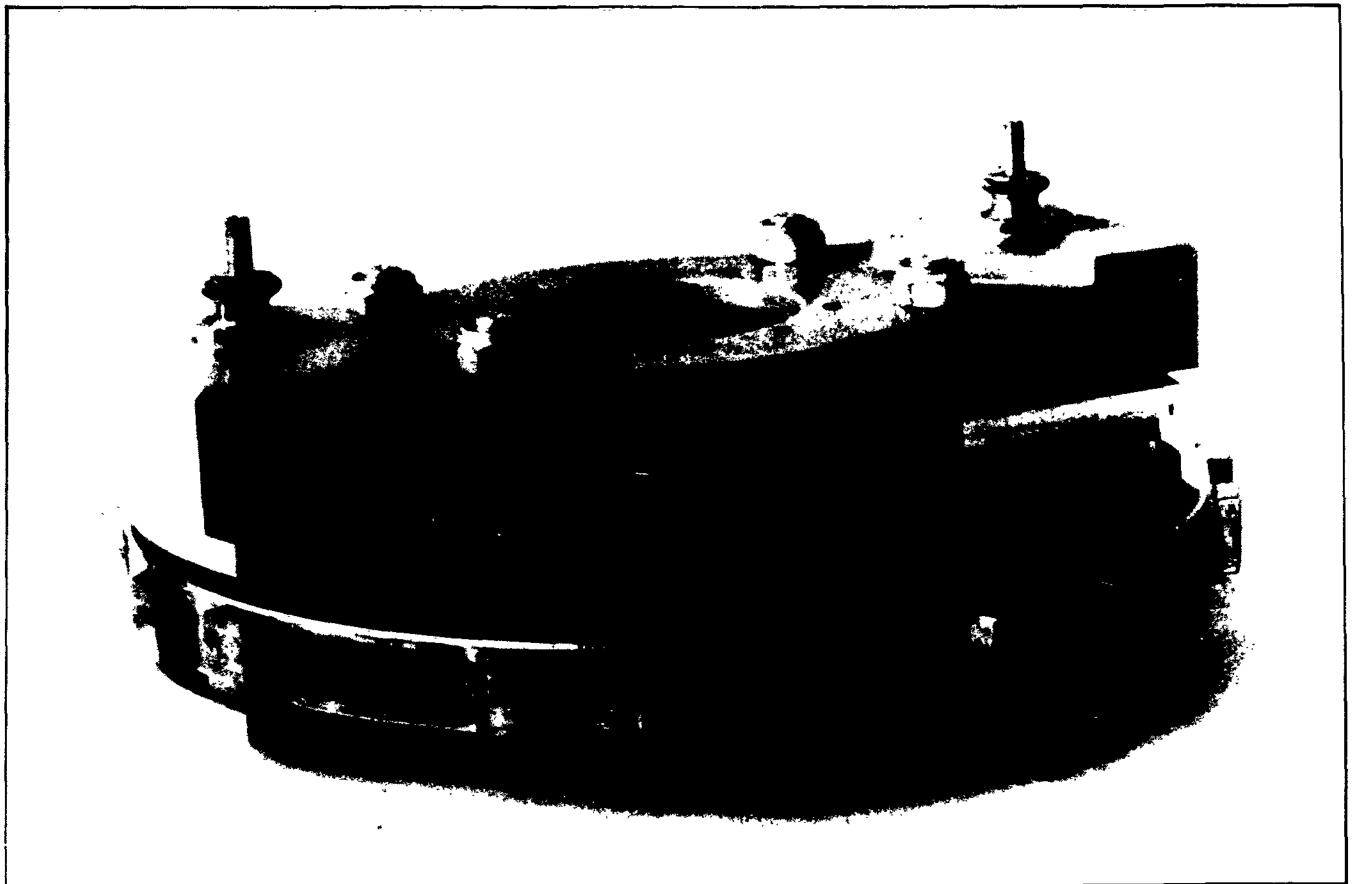
FIG. 11.



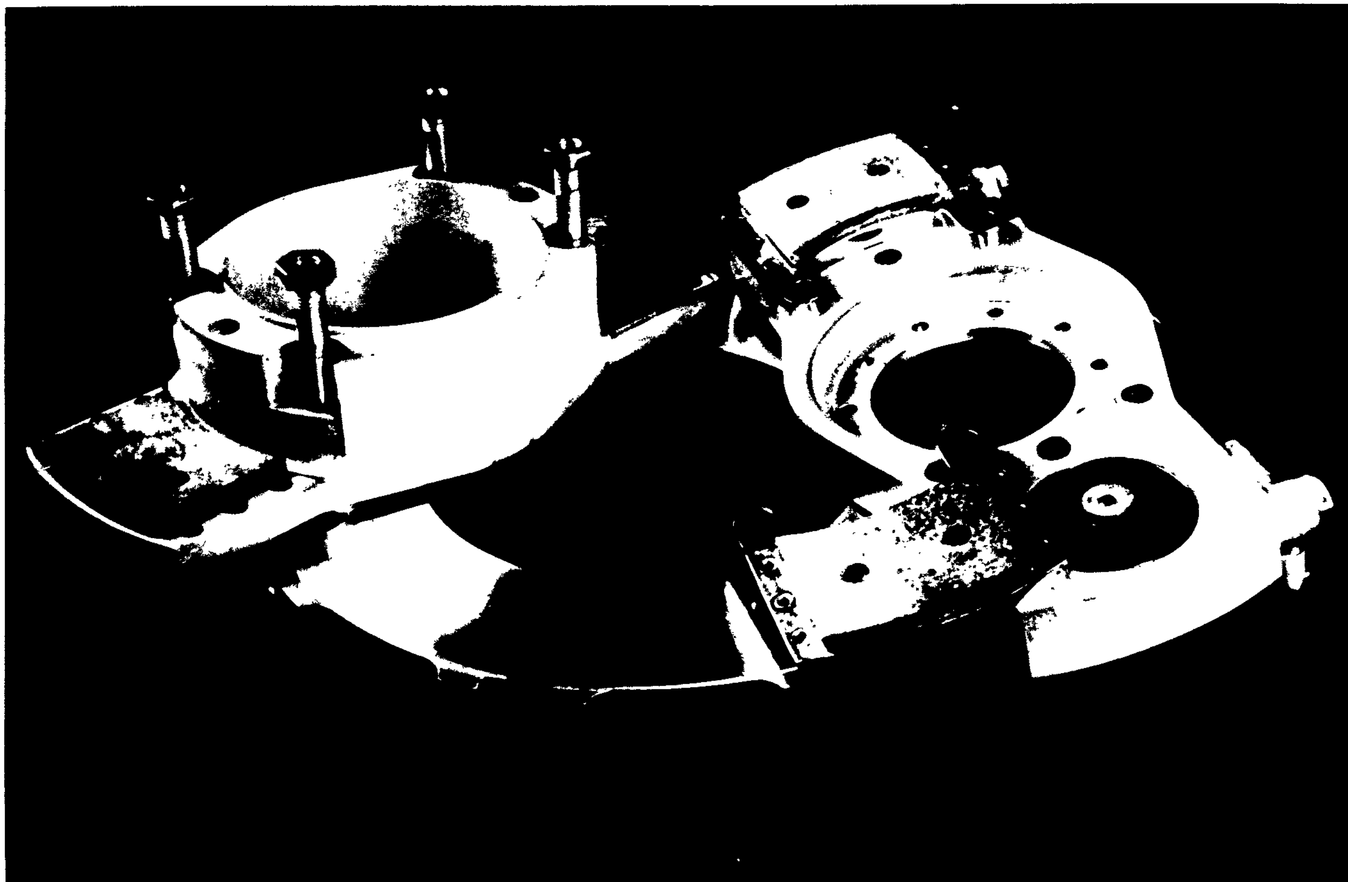
BRAKE D ASSEMBLY.

FIG. 12

BRAKE D. FRICTION ELEMENT FIG.13.



BRAKE E ASSEMBLY. FIG. 14.



BRAKE 'E' FRICTION ELEMENT. FIG. 15.

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AD#: AD029523

Date of Search: 30 April 2008

Record Summary: DSIR 23/22738
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